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THE LAW OF PARSIMONY.

"OCCAM'S RAZOR" is usually quoted in the form *Entia non sunt multiplicanda praeter necessitatem,* but it seems to be doubtful whether the "invincible doctor" actually used these words.[1] If he did not, however, he used others very similar, as for example, *Frustra fit per plura quod potest fieri per pauciora*; and since the "razor" is commonly used nowadays without special reference to

[1] See W. M. Thorburn, *Mind,* N. S., No. 94, pp. 287-288. Mr. Thorburn gives many references to the text of Occam (or Ockham), and also cites Hauréau (*Philosophie scholastique,* Chap. XXVIII), Erdmann (*History of Philosophy,* I, s. 216), De Wulf (*Mediæval Philosophy,* s. 368), and Prantl (*Geschichte der Logik,* pp. 327-340). The English reader will find a general account of Occam in *The Catholic Encyclopædia,* Vol. XV, p. 636, in Stöckl's *History of Mediæval Philosophy* (T. A. Finlay's translation, pp. 422-427), and in the late Dr. T. M. Lindsay's article on Occam in the *Encyclopædia Britannica* (11th ed., Vol. XIX). Mr. C. Delisle Burns wrote an article entitled "William of Ockham on Continuity" (*Mind,* N. S., No. 100), calling attention to certain parallels between Occam's views on continuity and those of Mr. Bertrand Russell.

A characteristic note of Sir William Hamilton's on the history of the Law of Parcimony (this spelling is sometimes preferred) may be mentioned in this connection. Hamilton says that the Law of Parcimony should be expressed as follows: "Neither *more* nor *more onerous* causes are to be assumed than are necessary to account for the phenomena." Then he continues:

"This phasis of the law has long been fully promulgated. Every condition of any consequence has, indeed, been articulately laid down by Aristotle; and subsequent philosophers, among whom Galen is conspicuous, have only repeated what was so clearly and so frequently inculcated by the Stagirite. Not only is it a maxim of his philosophy that God and Nature never operate without effect (*οὐδὲν μάτην, οὐδὲν ἐλλειπῶς ποιοῦσι*), they never operate superfluously (*μηδὲν περίεργον—περιττῶς—ἄργως*), but always through one rather than through a plurality of means (*καθ' ἓν μᾶλλον ἢ κατὰ πολλά*). The scholastic axioms: *Principia non sunt cumulanda; Frustra fit per plura quod fieri potest per pauciora; Natura horret superfluum,* these simply embody Aristotle's dicta; and the same, with a modification, is manifest of the *Novaculum Nominalium* or Occam's razor: *Entia non sunt multiplicanda praeter necessitatem.* Newton's first and principal rule of philosophizing, in so far as it is accurately expressed, simply repeats Aristotle's law: *Effectuum naturalium causae, non plures sunt admittendae quam quae, et verae sunt, et effectibus explicandis sufficiunt"* (*Discussions on Philosophy,* 3d edition, p. 624).

the scholastic theory of entities or universals, the precise form in which Occam stated it is irrelevant.

Among modern writers the principle is cited in very various connections. Sometimes it is regarded merely as a rule of method in the judicious choice of hypotheses. Scientific explanation cannot dispense with hypotheses, but it should be as sparing of them as it can. The principle of parsimony, in this sense, is therefore but a "safety razor" designed to protect against the dangers besetting the use of an indispensable instrument. On the other hand, the law of simplicity or economy in thinking is sometimes acclaimed as if it provided some sort of absolute guidance, and is even treated as the very essence of science, and therefore, one would suppose, of knowable reality. The design of this paper is to consider the principle both in its narrower aspect and in the more extended applications of it. As a preliminary to this inquiry, some general reflections concerning the principle and its difficulties seem to be required.

The maxim itself is naturally understood as a general appeal for economy couched in negative language denouncing extravagance. General appeals for economy have been common from the days of Isaiah the son of Amoz, but it is hard to see why a sumptuary edict of this sort should be more effective in matters of thinking than in matters political. Even when the need for economy is sincerely recognized it is never easy to determine where the path of true economy lies, for the simple reason that waste, luxury, and extravagance are relative terms. Why, then, should things be otherwise in this affair of thinking? Every one realizes that it is foolish to be recklessly prodigal of assumptions. That way incapacity lies. But in particular cases it is usually very difficult to prove that any given assumption is necessarily superfluous in every regard, very arbitrary to determine which regard is to be preferable to

all others, and entirely impossible to legislate *a priori* on the question.

None the less there is a disposition in certain quarters to apply the principle of parsimony to scientific investigations in a fashion that is neither merely negative nor merely regulative. Dr. Whitehead, for example, while disclaiming any knowledge of what he calls the metaphysical validity of Occam's razor, claims that the maxim has its roots in something deeper than logical elegance, and that it is obviously valid in scientific inquiries since "every use of hypothetical entities diminishes the claims of scientific reasoning to be the necessary outcome of harmony between thought and sense-presentation. As hypothesis increases, necessity diminishes."[2]

Mr. Russell, again, although he states in one place that when science selects the simplest formula that will fit the facts "this, quite obviously, is merely a methodological precept, not a law of nature,"[3] expresses himself at other times in a way that conveys a different suggestion. "By the principle of Occam's razor," he says, "if the class of appearances will fulfil the purposes for the sake of which the thing was invented by the prehistoric metaphysicians to whom common sense is due, economy demands that we should identify the thing with the class of its appearances";[4] and he goes on to state in the next paragraph that "the supreme maxim in scientific philosophizing is this: *Whereever possible, logical constructions are to be substituted for inferred entities.*"

These statements, it is true, are not directly contradictory since the second of them professes to be concerned only with a supposed hypothesis of prehistoric metaphysicians, and since Mr. Russell adds that "it is not necessary

[2] *The Organization of Thought*, pp. 175-176.

[3] *Mysticism and Logic*, p. 204.

[4] *Ibid.*, p. 155.

to *deny* a substance or substratum underlying these appearances; it is merely expedient to abstain from asserting this unnecessary entity."[5] At the same time the difference of emphasis between them is very notable and shows that there is room for inquiry concerning the limits within which Occam's razor can be applied in scientific theory. Still wider fields for discussion appear when the principle of intellectual economy receives the interpretation which Mach and others have given it. Mach affirms that the economical office of science is really its ultimate function, and claims that the full recognition of the economy of scientific thought is at the same time the abandonment of mystery in science.[6] He defines physics as "sense-experience economically arranged."[7] This view, in some form, is characteristic of the Kirchhoff school, and of the theories of many of the pragmatists. It will be considered in the later part of this paper. The earlier part will deal with the application of Occam's razor to the theory of hypotheses.

It is usual to distinguish between hypotheses concerning a fact and hypotheses concerning laws. The first class includes all suggested answers to such questions as "How did Cleopatra die?" or "Who stole the portrait of Mona Lisa?" Such hypotheses are attempts at reconstruction whose aim is the discovery of a single unique chain of events. Facts come to us piecemeal, but we cannot believe that they occurred piecemeal, and we endeavor to trace their history. In doing so we presuppose a certain constancy and continuity in nature, and the tracing of this continuity and constancy is quite as essential a part of scientific investigation as the task of generalizing from given data.

[5] *Ibid.*

[6] See, e. g., *The Science of Mechanics* (Eng. translation), pp. 481ff.

[7] *Populärwissenschaftliche Vorlesungen*, p. 219.

The main condition of the problem in attempting this kind of explanation is that the events which happened had just one history. To discover this history it is necessary to piece together such facts as are known to have occurred, and to frame conjectures concerning the gaps in the story which have not yet been ascertained. It might seem, then, that there is no room in such cases for any application of the principle of parsimony, and indeed that simplification of any sort is the chief error to guard against. Indeed, all the famous detectives of modern fiction explain that they owe their success, under Providence, to the fact that they are less liable than the police or than their biographers to be blinded by the glamor of the simplest theory. They find that it pays to follow every clue to its subtlest ramification.

This criticism, however, would plainly be mistaken. Simplification can never be correctly interpreted to mean the neglect of relevant data. Again, the fact that the events under investigation can have had but one history (which therefore simply *was* without any question of simplicity or the reverse) is irrelevant with regard to hypotheses concerning the way in which these events may be supposed to have happened before their history was traced. It is easy enough to make extravagant assumptions in these cases, and it is frequently a work of genius to discover the one correct explanation where so many are possible on the evidence. Moreover, it is clear that if the "correct" explanation is only probable and not quite certain, the principle of parsimony enters. A man is seen to go into a house. Two hours later he is observed to leave it wheeling a heavy load. If the load proves to be a corpse recently strangled, it will not avail the man to urge that the murder was done by an unknown assailant who entered and made his escape during that period. It may indeed be impossible to prove that there was no such as-

sailant, but the jury do not require this hypothesis. They employ the principle of parsimony. In this case, however, their hypothesis deals with what they suppose to be a sufficient cause of the event, and it is with regard to causes (and more generally to correlations) that the law of parsimony is usually employed.

According to Mach and the Kirchhoff school the laws of nature are merely concise general descriptions of phenomena, and the aim of science is to discover the most compendious descriptions of this kind. These statements, however, are somewhat misleading since they seem to ignore the characteristic function of induction. Induction is not mere description, but essentially a process of inference. It is inference from the observed to the unobserved, and the ultimate logical problem of induction is just the question how far this inference from given data to unobserved cases is valid. The most striking instance of this logical problem is the question of the validity of prediction, but precisely the same problem occurs in retrospective inductive inference.

This definition, however, might be amended to meet such criticisms, and causal laws might be interpreted to mean the general descriptions of a class of relations holding between phenomena. Induction involves generalization and is the process whereby, when certain relations have been ascertained to hold between a given number of observed instances without contrary instances, the number of such observations is deemed sufficient to justify the inference that it is probable that these relations hold in every instance of the occurrence of phenomena of the kind at any time or place. The class so described is not an empirical collection of observed cases. On the contrary only a few of its members have been observed. But on the strength of the observation of a few instances the inference is drawn that certain of the characteristics found in these

instances may be presumed to be true of every instance of the kind whether this instance is observed or unobserved, and, indeed, whether or not it is empirically possible to observe the instance. Such laws are also held to be true hypothetically when nothing of the sort occurred. No one doubts that Socrates would have died if he had stabbed himself to the heart before the time came for him to drink the hemlock, although, in point of fact, he preferred to await the due execution of the law.

Hypotheses concerning laws, therefore, imply a process whereby the analysis of a group of observed cases is extended by analogy to any member of the whole class of such cases. This procedure, it is plain, is connected with a principle of simplification. The analysis itself is selective since it endeavors to apprehend and retain the salient and relevant connections common to the observed cases, neglecting the others. Selective analysis is therefore in one of its aspects a process of simplification. Again there is simplification in the mere fact of dealing with a class in place of its several members. The definition of the class-concept supplies a rule for dealing with particular cases as need arises. It does not require the investigation of each particular instance. Indeed, when the number of members of the class is either indefinite so far as our information goes, or actually infinite, there is no chance of undertaking the detailed separate investigation. Moreover, the universal relationships common to the members of any class do not exhaust the individual characteristics of those members. There are limits set to generalization in the nature of things. The complexity of phenomena, in one sense, must simply be accepted. Nothing can make things less complex than they are; and the goal of scientific investigation is to select certain broad principles of relation, highly abstract and simple in themselves, which permit of

precise deduction according to a rule. This is the kind of knowledge which brings power with it.

There seems, however, to be no royal road to the kind of simplicity which is required, and the circumstance that a universal is always abstract when compared with a particular does not warrant any conclusion concerning which universals are most fundamental in any given case, or what universals can be safely neglected. It is not at all obvious, for instance, why the insight of scientific thinkers should have seized, during so many centuries, upon the primary qualities to the neglect of the secondary. Even granting that the primary qualities are the meeting-place of many diverse sensations, it is, to say the least, curious that they should have been so persistently supposed to be the sole characteristics of objective reality. The kind of simplicity most useful for science must be known by its fruits, and not on account of any antecedent argument concerning simplicity, except in respect of the quite general requirement that in proportion as a hypothesis is capable of being elaborated deductively it is likely to be useful for explanatory purposes.

It should be noted, moreover, that the abstract simplicity of general relations is not the only element required in scientific explanations. Mach says, it is true, that "continuity, economy, and constancy mutually condition one another," and that "they are really only different aspects of one and the same property of all sound thinking."[8] This statement is an excellent example of illegitimate simplification. It alleges that three distinct factors are aspects of one of the factors. Surely it is clear that there may be a very simple set of logical relations between isolated and momentary phenomena. The demand for continuity and constancy in the world, particularly as the medium

[8] *The Analysis of Sensations* (Eng. translation, ed. 1914), p. 328.

within which causal laws hold, is therefore quite distinct from the demand for simplicity.

If the sole aim of science were the discovery by analysis of the logical relations between phenomena, a large number of suggestions in scientific works would have, at best, the value of mere illustrations. For instance, the desire to suggest or construct a model acting according to any given formula would have no scientific justification. When the equations have been discovered, where is the need for a model? In this view also, there would be little meaning in Hertz's conception of physics according to which the object of that science is the construction of an image representing the behavior of the phenomena in the sense that the logical deductions from the character of this image agree with the observed results. Model and image alike are as particular as the observed phenomena. Moreover, both of them contain irrelevant features since neither image nor model can be *mere* instances of the logical relations expressed in the formula. Such images or models, therefore, would be nothing but aids to the imagination.

In that case it is hard to understand why so much attention should have been devoted to them. Why should Faraday's suggestions of this kind, or Lord Kelvin's, be accounted works of genius, and why should a physicist of Schuster's eminence regard any theory which confines itself to logical analysis as an evasion of the task of science?[9] Why should Newton have argued as he did concerning *actio in distans*? Newton, as is well known, stated in his *Regulae Philosophandi* that he refrained altogether from making hypotheses concerning gravitation. He confined himself to the laws which could be deduced by analysis of the phenomena. But he also denied strenuously that *actio in distans* could be interpreted as a *vera causa*, saying that

[9] "This evasive school of philosophy," Schuster, *The Theory of Optics*, Preface.

this was so great an absurdity that he believed "no man who has in philosophical matters a competent faculty of thinking can ever fall into it." "Gravity," he went on to say, "must be caused by an agent acting constantly according to certain laws."[10]

If the bodies of scientific theory are regarded as *foci* within a field, this view of Newton's can scarcely seem evident. The necessity for it was due to the conception that physical bodies, like the "things" of common sense, are defined by their enclosure in a limited spatial area, the consequence being that *actio in distans* was a plain negation of any real continuity in causal action. It was necessary, therefore, that there should be a common agent for things correlated according to the formula of gravitation, and a connecting medium for the action of this agent. On these assumptions Newton's argument was just, and the distinction he drew between an agent acting uniformly and a uniform formula of correlation very noteworthy. He regarded the discovery, through analysis, of a correlation in the phenomena as a radically incomplete explanation however important the correlation might be. Moreover, he thought that a full account of the kind of substance acting according to the formula, and of the continuous presence and operation of this substance, was required.

It might be argued, it is true, that the observed phenomena themselves are the only substances in the case. These observed facts, however, are far too few and too scattered to permit this interpretation. Whatever happens in the world must be regarded as part of the continuous connected whole, and the mere fact of correlation between certain fragments of this whole, arbitrarily selected or subjectively revealed, is not enough for the purposes of explanation. Other entities than the observed facts, on the one hand, and the formulas of connection, on the other,

[10] Letter to Bentley, Feb. 25, 1692-1693.

must be supposed. These entities must often be hypothetical since they cannot be verified by ordinary methods, and it is not likely that there is only one set of hypothetical entities which can be conceived to meet this legitimate demand for continuity. The principle of parsimony, therefore, if it is really fundamental, should be fitted to decide why a preference should be given to one class of these entities rather than to another.

As we have seen, Mr. Russell maintains that it is. The most usual theory of the physical world, he shows, is derived from the common-sense notion of a "thing." Common sense speedily substitutes the notion of permanent things in space for the series of sense-data, fluctuating and varying with each individual percipient, which are actually observed. Thence arises the theory that the material universe consists of bodies possessing only primary qualities and occupying definitive positions in absolute space at definitive moments of absolute time. The actual data of sense are subsequently explained as resultants of the action of these physical objects upon a certain set of physical objects which we call sense-organs and the brain, peculiarly connected with the mind.

Mr. Russell holds that this hypothesis is an offense against the principle of parsimony. In the first place, physical objects as thus conceived are quite unverifiable. What we perceive necessarily consists of sense-data; and these, on account of the familiar arguments proving the relativity of sense-data to the percipient, cannot be identified with physical objects. In the second place, these hypothetical physical objects are supposed to cause our perceptions, but this supposition ignores one of the cardinal rules in the discovery of causes. If A can be inferred to be causally connected with B, both A and B must have been frequently observed together without any contrary instances having been discovered after a stringent search for them. Accor-

ding to the usual theory, however, this supposed cause is never observed and never can be observed. It is only the hypothetical correlate of certain observed facts. Thus, by all the rules, this inference is an exceedingly precarious one, and these hypothetical correlates should be abandoned if possible.

Mr. Russell, accordingly, brings forward an alternative theory according to which physical objects are certain series of perceived sense-data and of unperceived *sensibilia*. The weakest part of this important theory, however, seems to be its reliance on the principle of parsimony. Even in terms of this principle, support for the new hypothesis could be claimed in two cases only. Either the new hypothesis is simpler than the old in all respects, or it is simpler in the most important respects. The first of these claims, as will speedily appear, is quite untenable. The second is always open to the objection that the standard of importance must be arbitrary and subjective.

No theory of the nature of the physical world can avoid making some unverifiable assumptions, and Mr. Russell's theory is certainly not exempt from this inevitable fate. His theory, in its present form, requires him not only to accept the present and the remembered sense-data of some individual experient, but also to postulate the existence of the present and remembered sense-data of other experients. and an immense range of *sensibilia* which are not actually sensed at all and can only be described as possible sense-data. The first of these postulates may be accepted without challenge. The second, however, calls for comment.

The present and remembered sense-data of all human percipients do not form a world, but merely scraps of a world; and the reason why this or the other percipient should perceive this or that at any given time requires the assumption of continuous connection in this world. The mere existence of sense-data is not self-explanatory. Harris

perceives a plate and a letter at breakfast time. He wants to know how the collection of sense-data which he calls a letter comes to be there, and he can scarcely do this without believing in a regular postal service. Again he is bound to reflect on the question how the same or a very similar collection of sense-data, called a plate, fulfils the same office for him every morning, and what he means by saying that the plate is on a shelf in his pantry when he and his household are asleep, and his cat is absent on a nocturnal ramble. To interpret these and similar experiences, Harris must be assured of the truth of a large, and, indeed, of an infinite number of hypothetical propositions. The plate, he knows, would look so and so in such and such a light; it would be part of an indistinguishable blur if he spun it on the table; it would look very different if he stood on his head and watched it; and if he were intoxicated or very sleepy it would assume all sorts of strange appearances.

The great advantage of the ordinary theory of perception is that it does not require us to assume the existence of any appearances except those which are actually presented to some percipient as the result of the stimulation of his sense-organs. These bizarre appearances, therefore, need not be supposed to exist except when Harris perceives them, and their peculiarities can then be explained by the state of Harris's nervous system. On the other hand, any theory which considers itself at liberty to assume the existence of unperceived *sensibilia,* the same in character as perceived sense-data, whenever there is a proved lack of continuity, suffers from the defect that it cannot logically set bounds to the number of these *sensibilia.* Like the magician in the fable it has made the well yield water, and cannot stop the flow. If it is legitimate (as, according to Mr. Russell's theory, it is necessary) to argue that the plate is not merely the circular disk which Harris per-

ceives when he looks down on it, but that it is also, at the very same time, the whole multitude of ellipses which he would perceive from this or the other angle, it must also be legitimate to argue that it is, in addition, any appearance which he would perceive if he were dazed or drunk. Moreover, Harris's dog, and cat, and canary, and the wasp on his window pane have a right to be consulted. If the plate at any time is the whole collection of colored shapes which Harris or any other man would perceive from any possible point of view, it should also, by the same argument, include what any other animal would perceive. It may well be doubted, therefore, whether a theory requiring such an enormous number of unperceived sensibles is any simpler than the one it is designed to supplant. Of course, if a logical construction can always be devised to replace an inferred entity, then any inferred entity in physics can be replaced by a logical construction. A logical construction, however, neither exists nor is a cause. If the whole realm of existence were composed of the sense-data of those human beings and other animals which have existed up to the present, and if these sense-data were the only actual causes in the world, then there would be no room for the inferred entities of current physical theory. But no one's sense-data can be explained without assuming the existence of other entities than these sense-data. The choice is, therefore, between inferring entities of the type of atoms and molecules, and inferring the existence of an enormous number of unperceived sensibles which yet are supposed to exist and to operate in precisely the same fashion as the perceived sense-data. This hypothesis of concealed sensibles is possibly the better of the two. But it cannot be said to be required by the principle of parsimony.

We may conclude, therefore, that general appeals to the principle of parsimony cannot settle questions of this character, a fact which is an important comment upon the

limitations of the principle of parsimony even when regarded as a rule for the choice of hypotheses. A larger issue remains, however. We can scarcely say, indeed, as Dr. Slop remarked to Corporal Trim on a celebrated occasion, "'Tis only a hypothesis, honest man. There's not a word of truth in it." A hypothesis claims to be probably true. But the general question of the relations between truth and intellectual economy raises still more fundamental problems than any considered hitherto.

On this larger issue there is a strongly marked antithesis in current philosophizing. On the one hand, it is common to find express statements to the effect that some particular theory is brought forward because it is simpler than its rivals, and not because it is therefore truer. It has the justification of a policy, not of a creed. On the other hand, truth itself is often defined as a species of intellectual convenience.

The stock example in connection with the former contention is the Copernican theory. Copernicus put forward the theory of his *De Revolutionibus* ostensibly as a mere hypothesis to simplify the task of saving the appearances. This device, however, was common in his time, and, indeed, was necessary for those who desired to live at peace with the Church. It is unlikely, therefore, that he regarded his conclusion as only an ingenious speculation. If he did, it is fair to conjecture that the grounds for his diffidence lay in the fact that the new hypothesis itself did not save the appearances without some cumbrousness and inexactitude. The principal reason for this, as modern readers can see, is that the retention by Copernicus of the postulate of uniform circular motion involved him in unnecessary difficulties and required him to presuppose some of the epicyclical machinery of Hipparchus and of Ptolemy's *Almagest*.

However that may be, Copernicus laid the foundation of the new theory securely in his proof of the relativity of

observable motion. When this had been done, the common sense of mankind began to believe not merely in the simplicity but in the truth of the theory. This seems eminently reasonable, since the inductive problem itself may very well be defined as the endeavor to discover the principles which, with the fewest possible assumptions, suffice to save the appearances. Accordingly, if induction yields truth it seems necessary to assume at least that an explanation of this kind which is simpler than some other is also more probably true. To affirm that any suggested explanation *is* true *simpliciter* is, of course, another matter.

At the same time, there is a serious difficulty even in this qualified statement. It seems to be plain that among formally correct demonstrations of the same conclusion in mathematics or in formal logic, one may be simpler than another while both must be equally true, just because both are true. More generally, in the formal deduction of pure mathematics or symbolic logic there is a choice of sets of primitive propositions from which the system itself may be deduced. While there may be greater convenience and elegance in choosing one of these sets of primitive propositions in preference to the others, there can scarcely be any difference in respect of truth.

It is true that part of the difficulty thus arising may be surmounted readily enough. When Schopenhauer complained that many of Euclid's demonstrations were only conjuring tricks, he meant, presumably, that Euclid had recourse to special artifices and minor principles in his proofs although these artifices really depended upon more general ones. The difficulty, in fact, would be overcome if there were only one set of ultimate general principles upon which all others depended; but this solution is plainly inadequate if there are alternative sets of such fundamental propositions. The same conclusion follows with reference

to any system of scientific concepts. Mach[11] remarks, very truly, that in all sciences a formal stage of development must succeed the deductive stage, since the principles assumed from time to time during the growth of the science must be revised and clarified and shown to be parts of a single harmonious elegant system. If, however, this formal development may take place on alternative lines, it does not seem possible to claim greater truth for the simpler system.

The contention that all intellectual thinking (and, ultimately, truth itself) is nothing but an economical contrivance, requires more elaborate consideration since there are so many different ways of interpreting it.

The most usual way of putting the case, is to affirm that thinking consists essentially in the use of symbols, and that symbols are merely abbreviated substitutes for facts. This is the substance of Mach's arguments.[12] Language, he says, is but an economic contrivance. Concepts are only compendious symbols for a fluid mass of fact. The concept of a "thing," for instance, is an abbreviated sign for a compound set of sensations of relative fixity. The concept of cause and effect is the selection of what purports to be a recurrent process in nature, although the fact that nature is present only once in any event shows that there can be no real recurrence. Similarly the laws of nature are short simple formulas standing for a multitude of facts. Finally, mathematics, the great instrument of the sciences, is itself the economy of counting; and counting is an economy.

These arguments are plausible until they are examined, but examination reveals a fatal weakness in their lax use of the term "symbol." Mach's reference to language, and the rest of his argument, suggest that he means by a

[11] *The Science of Mechanics*, p. 421.

[12] See, e. g., *The Science of Mechanics*, pp. 481ff.

"symbol" the conventional sounds of oral communication or the marks on paper that constitute a mathematical notation. It is plain, however, that thinking does not consist in making these noises or marks. These are only a device for expressing the thinker's meaning. What, then, is the meaning expressed in these ways?

The answer usually given is that a thinker's meaning is a series of concepts which, in their turn, symbolize or refer to certain facts. The problem turns, therefore, on the interpretation of concepts, and here also many ambiguities lurk. A concept is sometimes described as an abstraction from the facts, as if it were a residuum extracted or distilled from them. Again, it is sometimes described as a schematic mental image which can be used as a substitute for the facts. It is clear, however, that neither of these descriptions is adequate. If a condensed extract from the facts could, by any ingenuity, be conceived to stand for the rest of them, this would only be possible if both were instances of a common universal, and if a truncated or schematic psychological image (supposing such a thing to exist) stands for a class of objects, the reason again must be that this image exemplifies a universal term or relation which holds of the objects. In the latter case the image is a pictorial substitute for thought. But there can be no signification except through universals, and what is usually called a concept is nothing but the universal itself. True intellectual thinking is the discovery of universals which hold of particulars, and the universals so discovered are not psychological artifacts and are not a whit less objective or a whit more arbitrary than particular existent facts.

Sometimes, however, an attempt is made to parry this argument by interpreting concepts or universals in a different fashion. Knowledge, instead of being regarded as a process of discovery, is said to be a kind of organization,

and universals are but a means of organizing experience. The goal of thinking, according to this theory, is the attainment of harmony between intellectual processes and sense experience; and this harmony presupposes a certain simplicity in the principles according to which the intellect works *de facto*.

This argument itself has different meanings according to the philosophy of the users of it. The most obvious interpretation is the psychological one chosen by the pragmatists. According to this view the harmony desired in science is only a psychological harmony between two classes of mental facts, sensations and intellectual processes. It is a kind of psychological convenience. This statement, however, plainly requires a definition of the particular kind of harmony which is sought in logical thinking, since there are several alternative ways of attaining psychological harmony.

Psychological harmony, indeed, may be reached comparatively easily. The constitution of our minds has seen to that. Most people obtain it in these matters by combining association, hearsay evidence, and emotions with, here and there, a little dose of thinking. Indeed, the most convenient way of dealing with recalcitrant phenomena or with contradictions in points of theory is usually to forget them. This is the course adopted by ninety-nine minds out of a hundred, and it often works well enough although it shuts the door to all discovery and to any real advance even in matters of practice. How is it at all convenient to ponder over a trifling difficulty of forty seconds in a century in the case of the perihelion of Mercury or to reflect deeply on the absence of shifting of interference bands in the Michelson-Morley experiment? The motive for debating these problems is the desire for understanding, not the desire for convenience. The scientific mind, in a word, differs from other minds precisely because it finds

grounds for inconvenience and worry where the others find none. In the long run, it is true, the sustained reflection on these inconvenient problems may lead to greater convenience in the way of clear thinking and control over nature than the complacent blindness of common sense. But this is not the psychological motive for the investigation. That motive is truth and not ease.

A similar conclusion follows when this harmony of experience is interpreted in the very different fashion which such writers as Mr. Bradley have made familiar. It is tacitly presupposed in these theories that the harmony of experience is of a logical type, even if, as in the metaphyics of Plotinus, Spinoza, or Mr. Bosanquet, the work of the logical intellect must be superseded, "in the end," by the more comprehensive and satisfying unity of intuition or mysticism. According to this metaphysic, it is true, our minds are only fragments of a cosmic whole, so that it is impossible to argue directly from the facts of human psychology to that cosmic ideal which is also the truth. On the other hand, the only clue to the nature of the Absolute is human experience "at its best"; and the phrase "at its best" in this connection must mean "at the most stringent level of logic." The harmony in question, therefore, is a very special sort of harmony.

None the less, a metaphysical basis for the law of economy would seem to be established if either of the above theories could be accepted. If truth consists in a particular kind of harmony of experience, and if concepts are particular ways of organizing experience with simplicity as part of their essence, then the law of economy is part of the being of truth. It would be necessary, indeed, for Mach or the pragmatists carefully to distinguish the sort of organization which we call logical thinking from other species of mental organization such as sentiment or association. The Absolutists, also, would require to define

more narrowly what particular species of harmony it is which gives legitimate intellectual satisfaction. But the cosmic experience which, according to them, contains the whole truth, of which scientific generalizations are only faint adumbrations, must, one would suppose, be constituted according to certain organizing principles of supreme generality which are simple in outline and few in number. In our experience these are, *par excellence*, the organizing principles of our private worlds. They must also be presumed to be the organizing principles of that all-inclusive world in which science itself has its being.

There is an ultimate obstacle, however, in the way of accepting all such theories. The discovery of truth, it may be admitted, gives satisfaction and consequently indicates the presence of a certain psychological harmony. But this satisfaction is really due to the finality of the discovery. That is found which was sought, and the mind can rest in its discovery and gain confidence in its further powers of advancing. This harmony, then, is of quite a peculiar sort, and it presupposes the belief that a discovery has really been made. Truth is found and not manufactured.

The same argument holds of mental organization. The knowing mind must be regarded as an instrument capable of discovering a truth which in the last resort is simply given to it. This truth, however, is not given to it without infinite pains and labor, and the organization of the mind is the laborious process which the mind requires in order to attain the truth. The mind cannot ascertain the character of its objects without the organization of thought. It does not matter essentially whether these objects, in particular cases, are mental or not. The knowing mind has to discover the characteristics of sense-data in the same way as it has to discover anything else, and nothing that is known is a product of the knowing of it. The thinker, then, has to organize his mind in such a way as to make discovery

possible, and, in particular, to dwell upon, hold fast to, and follow up those universal principles and relationships which enable him to comprehend a multitude of particulars. Such universals are not psychological constructs or psychological forces. They are as objective as any particulars What we ought to do is to organize our minds in order to keep the right principles in view, and to discover more and more of their nature.

If these contentions are sound, it is very difficult, indeed, to find a metaphysical basis for any form of the law of economy, and the natural inference is certainly that the law is only a precept indicating the path of wisdom. No doubt there is a certain connection between simplicity and any true propositions which imply generalization or imply that a universal rule holds of a multitude of instances. On the other hand, the significance and importance of simplicity for us seems to be much greater than its significance for the world. Things are what they are in all their wealth of detail. Why, then, should the truth of things be less subtle? Thus it seems plain common sense to urge that there can be no *a priori* law to the effect that either a thing or the truth about it is simple. Is the fact of the matter anything more than the truism that the principles which we are capable of apprehending must be simple, and that we, in our own interests, should try to keep to the simplest of them?

This position rests on strong ground. The complexity of the universe can never be simplified out of existence. It remains on our hands always. We may discover certain laws of connection running through it, but these are only a small selection from its total character. It pays us to attend to these laws since no others are of any use to us. But it would be the veriest anthropomorphism to make our advantage a criterion of the character of true being. It is conceivable, therefore, that the law of economy is neither

more nor less than a rule of direction stating that we should always select the simplest of any general propositions which may be true and ascertainable, since it is these truths only that are likely to aid the mind in making further discoveries. This rule shows the path of wisdom in view of the limited powers and scope of the mind.

On the other hand, there is at least a possibility that this conclusion exaggerates the limitations of the human intellect, and that its very modesty leads to a sort of neurasthenic impotence instead of a structural one. The presupposition of the argument is that certain truths, both universal and particular, can be discovered, and it is scarcely doubtful that there is a fundamental order of precedence in the kingdom of knowable truths. The laws of logic, for example, take precedence of all others, and many humble generalizations must give the *pas* to the law of gravitation or to the conservation of energy. If, then, all truth is discovery, it is hard to suppose that this discovery should have preeminent importance in the nature of things, or that it should indicate nothing except the limitations of human minds. Would there not be a hierarchy of truths even for an omniscient knower, and would not the fundamental truths in this hierarchy be also simple? Is it conceivable, for instance, that the laws of logic would be less fundamental for such a being than they are for us?

It may be a biological accident that the logical intellect is capable of apprehending the truth of things in any measure. But this accident of evolution, if it is one, entails consequences; and if human beings can really understand at all, it is hard to suppose that the simplicity which is essential to the understanding has no genuine counterpart in reality. The fact that we *can* understand indicates the contrary, whatever the history of this power of comprehension may have been. On the other hand, this general conclusion can scarcely be used to decide particular cases,

and even the tentative suggestion that there must be an essence of reality, and that this essence must be capable of being resolved into a limited number of supremely general propositions from which the minor ones can be deduced, seems to go distinctly further than the evidence requires. The laws of logic, indeed, must be judged to occupy a unique position in reality as well as in thought, but none of the specific facts of existence can be deduced from them alone.

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THE PHILOSOPHY OF LOGICAL ATOMISM.

VII. THE THEORY OF TYPES AND SYMBOLISM: CLASSES.

BEFORE I begin to-day the main subject of my lecture, I should like to make a few remarks in explanation and amplification of what I have said about existence in my previous two lectures. This is chiefly on account of a letter I have received from a member of the class, raising many points which, I think, were present in other minds too.

The first point I wish to clear up is this: I did not mean to say that when one says a thing exists, one means the same as when one says it is possible. What I meant was, that the fundamental logical idea, the primitive idea, out of which both those are derived is the same. That is not quite the same thing as to say that the statement that a thing exists is the same as the statement that it is possible, which I do not hold. I used the word "possible" in perhaps a somewhat strange sense, because I wanted some word for a fundamental logical idea for which no word exists in ordinary language, and therefore if one is to try to express in ordinary language the idea in question, one has to take some word and make it convey the sense that I was giving to the word "possible," which is by no means the only sense that it has but is a sense that was convenient for my purpose. We say of a propositional function that it is possible, where there are cases in which it is true.

That is not exactly the same thing as what one ordinarily means, for instance, when one says that it is possible it may rain to-morrow. But what I contend is, that the ordinary uses of the word "possible" are derived from this notion by a process. E. g., normally when you say of a proposition that it is possible, you mean something like this: first of all it is implied that you do not know whether it is true or false, and I think it is implied; secondly, that it is one of a class of propositions, some of which are known to be true. When I say, e. g., "It is possible that it may rain to-morrow,"—"It will rain to-morrow" is one of the class of propositions "It rains at time *t*," where *t* is different times. We mean partly that we do not know whether it will rain or whether it will not, but also that we do know that that is the sort of proposition that is quite apt to be true, that it is a value of a propositional function of which we know some value to be true. Many of the ordinary uses of "possible" come under that head, I think you will find. That is to say, that if you say of a proposition that it is possible, what you have is this: "There is in this proposition some constituent, which, if you turn it into a variable, will give you a propositional function that is sometimes true." You ought not therefore to say of a proposition simply that it is possible, but rather that it is possible in respect of such-and-such a constituent. That would be a more full expression.

When I say, for instance, that "Lions exist," I do not mean the same as if I said that lions were possible; because when you say "Lions exist," that means that the propositional function "*x* is a lion" is a possible one in the sense that there are lions, while when you say "Lions are possible" that is a different sort of statement altogether, not meaning that a casual individual animal may be a lion, but rather that a *sort* of animal may be the *sort* that we call "lions." If you say "Unicorns are possible," e. g., you

would mean that you do not know any reason why there should not be unicorns, which is quite a different proposition from "Unicorns exist." As to what you would mean by saying that unicorns are possible, it would always come down to the same thing as "It is possible it may rain tomorrow." You would mean, the proposition "There are unicorns" is one of a certain set of propositions some of which are known to be true, and that the description of the unicorn does not contain in it anything that *shows* there could not be such beasts.

When I say a propositional function is possible, meaning there are cases in which it is true, I am consciously using the word "possible" in an unusual sense, because I want a single word for my fundamental idea, and cannot find any word in ordinary language that expresses what I mean.

Secondly, it is suggested that when one says a thing exists, it means that it is in time, or in time and space, at any rate in time. That is a very common suggestion, but I do not think that really there is much to be said for that use of the words; in the first place, because if that were all you meant, there would be no need for a separate word. In the second place, because after all in the sense, whatever that sense may be, in which the things are said to exist that one ordinarily regards as existing, one may very well wish to discuss the question whether there are things that exist without being in time. Orthodox metaphysics holds that whatever is really real is not in time, that to be in time is to be more or less unreal, and that what really exists is not in time at all. And orthodox theology holds that God is not in time. I see no reason why you should frame your definition of existence in such a way as to preclude that notion of existence. I am inclined to think that there are things that are not in time, and I should be sorry to use the word existence in that sense when you

have already the phrase "being in time" which quite sufficiently expresses what you mean.

Another objection to that definition is, that it does not in the least fit the sort of use of "existence" which was underlying my discussion, which is the common one in mathematics. When you take existence-theorems, for instance, as when you say "An even prime exists," you do not mean that the number two is in time but that you can find a number of which you can say "This is even and prime." One does ordinarily in mathematics speak of propositions of that sort as existence-theorems, i. e., you establish that there is an object of such-and-such a sort, that object being, of course, in mathematics a logical object, not a particular, not a thing like a lion or a unicorn, but an object like a function or a number, something which plainly does not have the property of being in time at all, and it is that sort of sense of existence-theorems that is relevant in discussing the meaning of existence as I was doing in the last two lectures. I do, of course, hold that that sense of existence can be carried on to cover the more ordinary uses of existence, and does in fact give the key to what is underlying those ordinary uses, as when one says that "Homer existed" or "Romulus did not exist," or whatever we may say of that kind.

I come now to a third suggestion about existence, which is also a not uncommon one, that of a given particular "this" you can say "This exists" in the sense that it is not a phantom or an image or a universal. Now I think that use of existence involves confusions which it is exceedingly important to get out of one's mind, really rather dangerous mistakes. In the first place, we must separate phantoms and images from universals; they are on a different level. Phantoms and images do undoubtedly exist in that sense, whatever it is, in which ordinary objects exist. I mean, if you shut your eyes and imagine some visual scene, the

images that are before your mind while you are imagining are undoubtedly there. They are images, something is happening, and what is happening is that the images are before your mind, and these images are just as much part of the world as tables and chairs and anything else. They are perfectly decent objects, and you only call them unreal (if you call them so), or treat them as non-existent, because they do not have the usual sort of relations to other objects. If you shut your eyes and imagine a visual scene and you stretch out your hand to touch what is imaged, you won't get a tactile sensation, or even necessarily a tactile image. You will not get the usual correlation of sight and touch. If you imagine a heavy oak table, you can remove it without any muscular effort, which is not the case with oak tables that you actually see. The general correlations of your images are quite different from the correlations of what one chooses to call "real" objects. But that is not to say images are unreal. It is only to say they are not part of physics. Of course, I know that this belief in the physical world has established a sort of reign of terror. You have got to treat with disrespect whatever does not fit into the physical world. But that is really very unfair to the things that do not fit in. They are just as much there as the things that do. The physical world is a sort of governing aristocracy, which has somehow managed to cause everything else to be treated with disrespect. That sort of attitude is unworthy of a philosopher. We should treat with exactly equal respect the things that do not fit in with the physical world, and images are among them.

"Phantoms," I suppose, are intended to differ from "images" by being of the nature of hallucinations, things that are not merely imagined but that go with belief. They again are perfectly real; the only odd thing about them is their correlations. Macbeth sees a dagger. If he tried to touch it, he would not get any tactile sensation, but that

does not imply that he was not *seeing* a dagger, it only implies that he was not *touching* it. It does not in any way imply that the visual sensation was not there. It only means to say that the sort of correlation between sight and touch that we are used to is the normal rule but not a universal one. In order to pretend that it is universal, we say that a thing is unreal when it does not fit in. You say "Any man who is a man will do such-and-such a thing." You then find a man who will not, and you say, he is not a man. That is just the same sort of thing as with these daggers that you cannot touch.

I have explained elsewhere the sense in which phantoms are unreal.[1] When you see a "real" man, the immediate object that you see is one of a whole system of particulars, all of which belong together and make up collectively the various "appearances" of the man to himself and others. On the other hand, when you see a phantom of a man, that is an isolated particular, not fitting into a system as does a particular which one calls an appearance of the "real" man. The phantom is in itself just as much part of the world as the normal sense-datum, but it lacks the usual correlation and therefore gives rise to false inferences and becomes deceptive.

As to universals, when I say of a particular that it exists, I certainly do not mean the same thing as if I were to say that it is not a universal. The statement concerning any particular that it is not a universal is quite strictly nonsense—not false, but strictly and exactly nonsense. You never can place a particular in the sort of place where a universal ought to be, and *vice versa*. If I say "*a* is not *b*," or if I say "*a* is *b*," that implies that *a* and *b* are of the same logical type. When I say of a universal that it exists, I should be meaning it in a different

[1] See *Our Knowledge of the External World*, Chap. III. Also Section XII of "Sense-Data and Physics" in *Mysticism and Logic*.

sense from that in which one says that particulars exist. E. g., you might say "Colors exist in the spectrum between blue and yellow." That would be a perfectly respectable statement, the colors being taken as universals. You mean simply that the propositional function "x is a color between blue and yellow" is one which is capable of truth. But the x which occurs there is not a particular, it is a universal. So that you arrive at the fact that the ultimate important notion involved in existence is the notion that I developed in the lecture before last, the notion of a propositional function being sometimes true, or being, in other words, possible. The distinction between what some people would call real existence, and existence in people's imagination or in my subjective activity, that distinction, as we have just seen, is entirely one of correlation. I mean that anything which appears to you, you will be mistakenly inclined to say has some more glorified form of existence if it is associated with those other things I was talking of in the way that the appearance of Socrates to you would be associated with his appearance to other people. You would say he was only in your imagination if there were not those other correlated appearances that you would naturally expect. But that does not mean that the appearance to you is not exactly as much a part of the world as if there were other correlated appearances. It will be exactly as much a part of the real world, only it will fail to have the correlations that you expect. That applies to the question of sensation and imagination. Things imagined do not have the same sort of correlations as things sensated. If you care to see more about this question, I wrote a discussion in *The Monist* for January, 1915, and if any of you are interested, you will find the discussion there.

I come now to the proper subject of my lecture, but shall have to deal with it rather hastily. It was to explain the theory of types and the definition of classes. Now

first of all, as I suppose most of you are aware, if you proceed carelessly with formal logic, you can very easily get into contradictions. Many of them have been known for a long time, some even since the time of the Greeks. but it is only fairly recently that it has been discovered that they bear upon mathematics, and that the ordinary mathematician is liable to fall into them when he approaches the realms of logic, unless he is very cautious. Unfortunately the mathematical ones are more difficult to expound, and the ones easy to expound strike one as mere puzzles or tricks.

You can start with the question whether or not there is a greatest cardinal number. Every class of things that you can choose to mention has some cardinal number. That follows very easily from the definition of cardinal numbers as classes of similar classes, and you would be inclined to suppose that the class of all the things there are in the world would have about as many members as a class could be reasonably expected to have. The plain man would suppose you could not get a larger class than the class of all the things there are in the world. On the other hand, it is very easy to prove that if you take selections of some of the members of a class, making those selections in every conceivable way that you can, the number of different selections that you can make is greater than the original number of terms. That is easy to see with small numbers. Suppose you have a class with just three numbers, *a, b, c.* The first selection that you can make is the selection of no terms. The next of *a* alone, *b* alone, *c* alone. Then *bc, ca, ab, abc,* which makes in all 8 (i.e., 2^3) selections. Generally speaking, if you have *n* terms, you can make 2^n selections. It is very easy to prove that 2^n is always greater than *n,* whether *n* happens to be finite or not. So you find that the total number of things in the world is not so great as the number of classes

that can be made up out of those things. I am asking you to take all these propositions for granted, because there is not time to go into the proofs, but they are all in Cantor's work. Therefore you will find that the total number of things in the world is by no means the greatest number. On the contrary, there is a hierarchy of numbers greater than that. That, on the face of it, seems to land you in a contradiction. You have, in fact, a perfectly precise arithmetical proof that there are *fewer* things in heaven or earth than are dreamt of in *our* philosophy. That shows how philosophy advances.

You are met with the necessity, therefore, of distinguishing between classes and particulars. You are met with the necessity of saying that a class consisting of two particulars is not itself in turn a fresh particular, and that has to be expanded in all sorts of ways; i. e., you will have to say that in the sense in which there are particulars, in that sense it is not true to say there are classes. The sense in which there are classes is a different one from the sense in which there are particulars, because if the senses of the two were exactly the same, a world in which there are three particulars and therefore eight classes, would be a world in which there are at least eleven things. As the Chinese philosopher pointed out long ago, a dun cow and a bay horse makes three things: separately they are each one, and taken together they are another, and therefore three.

I pass now to the contradiction about classes that are not members of themselves. You would say generally that you would not expect a class to be a member of itself. For instance, if you take the class of all the teaspoons in the world, that is not in itself a teaspoon. Or if you take all the human beings in the world, the whole class of them is not in turn a human being. Normally you would say you cannot expect a whole class of things to be

itself a member of that class. But there are apparent exceptions. If you take, e. g., all the things in the world that are not teaspoons and make up a class of them, that class obviously (you would say) will not be a teaspoon. And so generally with negative classes. And not only with negative classes, either, for if you think for a moment that classes are things in the same sense in which things are things, you will then have to say that the class consisting of all the things in the world is itself a thing in the world, and that therefore this class is a member of itself. Certainly you would have thought that it was clear that the class consisting of all the classes in the world is itself a class. That I think most people would feel inclined to suppose, and therefore you would get there a case of a class which is a member of itself. If there is any sense in asking whether a class is a member of itself or not, then certainly in all the cases of the ordinary classes of everyday life you find that a class is not a member of itself. Accordingly, that being so, you could go on to make up the class of all those classes that are not members of themselves, and you can ask yourself, when you have done that, is that class a member of itself or is it not?

Let us first suppose that it is a member of itself. In that case it is one of those classes that are not members of themselves, i. e., it is not a member of itself. Let us then suppose that it is not a member of itself. In that case it is not one of those classes that are not members of themselves, i. e., it is one of those classes that are members of themselves, i. e., it is a member of itself. Hence either hypothesis, that it is or that it is not a member of itself, leads to its contradiction. If it is a member of itself, it is not, and if it is not, it is.

That contradiction is extremely interesting. You can modify its form; some forms of modification are valid and some are not. I once had a form suggested to me which

was not valid, namely the question whether the barber shaves himself or not. You can define the barber as "one who shaves all those, and those only, who do not shave themselves." The question is, does the barber shave himself? In this form the contradiction is not very difficult to solve. But in our previous form I think it is clear that you can only get around it by observing that the whole question whether a class is or is not a member of itself is nonsense, i. e., that no class either is or is not a member of itself, and that it is not even true to say that, because the whole form of words is just a noise without meaning. That has to do with the fact that classes, as I shall be coming on to show, are incomplete symbols in the same sense in which the descriptions are that I was talking of last time; you are talking nonsense when you ask yourself whether a class is or is not a member of itself, because in any full statement of what is meant by a proposition which seems to be about a class, you will find that the class is not mentioned at all and that there is nothing about a class in that statement. It is absolutely necessary, if a statement about a class is to be significant and not pure nonsense, that it should be capable of being translated into a form in which it does not mention the class at all. This sort of statement, "Such-and-such a class is or is not a member of itself," will not be capable of that kind of translation. It is analogous to what I was saying about descriptions: the symbol for a class is an incomplete symbol; it does not really stand for part of the propositions in which symbolically it occurs, but in the right analysis of those propositions that symbol has been broken up and disappeared.

There is one other of these contradictions that I may as well mention, the most ancient, the saying of Epimenides that "All Cretans are liars." Epimenides was a man who slept for sixty years without stopping, and I believe that it was at the end of that nap that he made the remark that

all Cretans were liars. It can be put more simply in the form: if a man makes the statement "I am lying," is he lying or not? If he is, that is what he said he was doing, so he is speaking the truth and not lying. If, on the other hand, he is not lying, then plainly he is speaking the truth in saying that he is lying, and therefore he is lying, since he says truly that that is what he is doing. It is an ancient puzzle, and nobody treated that sort of thing as anything but a joke until it was found that it had to do with such important and practical problems as whether there is a greatest cardinal or ordinal number. Then at last these contradictions were treated seriously. The man who says "I am lying" is really asserting "There is a proposition which I am asserting and which is false." That is presumably what you mean by lying. In order to get out the contradiction you have to take that whole assertion of his as one of the propositions to which his assertion applies; i. e., when he says "There is a proposition which I am asserting and which is false," the word "proposition" has to be interpreted as to include among propositions his statement to the effect that he is asserting a false proposition. Therefore you have to suppose that you have a certain totality, viz., that of propositions, but that that totality contains members which can only be defined in terms of itself. Because when you say "There is a proposition which I am asserting and which is false," that is a statement whose meaning can only be got by reference to the totality of propositions. You are not saying which among all the propositions there are in the world it is that you are asserting and that is false. Therefore it presupposes that the totality of propositions is spread out before you and that some one, though you do not say which, is being asserted falsely. It is quite clear that you get into a vicious circle if you first suppose that this totality of propositions is spread out before you, so that you can

without picking any definite one say "Some one out of this totality is being asserted falsely," and that yet, when you have gone on to say "Some one out of this totality is being asserted falsely," that assertion is itself one of the totality you were to pick out from. That is exactly the situation you have in the paradox of the liar. You are supposed to be given first of all a set of propositions, and you assert that some one of these is being asserted falsely, then that assertion itself turns out to be one of the set, so that it is obviously fallacious to suppose the set already there in its entirety. If you are going to say anything about "all propositions," you will have to define propositions, first of all, in some such way as to exclude those that refer to all the propositions of the sort already defined. It follows that the word "proposition," in the sense in which we ordinarily try to use it, is a meaningless one, and that we have got to divide propositions up into sets and can make statements about all propositions in a given set, but those propositions will not themselves be members of the set. For instance, I may say "All atomic propositions are either true or false," but that itself will not be an atomic proposition. If you try to say "All propositions are either true or false," without qualification, you are uttering nonsense, because if it were not nonsense it would have to be itself a proposition and one of those included in its own scope, and therefore the law of excluded middle as enunciated just now is a meaningless noise. You have to cut propositions up into different types, and you can start with atomic propositions or, if you like, you can start with those propositions that do not refer to sets of propositions at all. Then you will take next those that refer to sets of propositions of that sort that you had first. Those that refer to sets of propositions of the first type, you may call the second type, and so on.

If you apply that to the person who says "I am lying,"

you will find that the contradiction has disappeared, because he will have to say what type of liar he is. If he says "I am asserting a false proposition of the first type," as a matter of fact that statement, since it refers to the totality of propositions of the first type, is of the second type. Hence it is not true that he is asserting a false proposition of the first type, and he remains a liar. Similarly, if he said he was asserting a false proposition of the 30,000th type, that would be a statement of the 30,001th type, so he would still be a liar. And the counter-argument to prove that he was also not a liar has collapsed.

You can lay it down that a totality of any sort cannot be a member of itself. That applies to what we are saying about classes. For instance, the totality of classes in the world cannot be a class in the same sense in which they are. So we shall have to distinguish a hierarchy of classes. We will start with the classes that are composed entirely of particulars: that will be the first type of classes. Then we will go on to classes whose numbers are classes of the first type: that will be the second type. Then we will go on to classes whose members are classes of the second type: that will be the third type, and so on. Never is it possible for a class of one type either to be or not to be identical with a class of another type. That applies to the question I was discussing a moment ago, as to how many things there are in the world. Supposing there are three particulars in the world. There are then, as I was explaining, 8 classes of particulars. There will be 2^8 (i. e., 256) classes of classes or particulars, and 2^{256} classes of classes of classes of particulars, and so on. You do not get any contradiction arising out of that, and when you ask yourself the question: "Is there, or is there not a greatest cardinal number?" the answer depends entirely upon whether you are confining yourself within some one type, or whether you are not. Within any given type there is

a greatest cardinal number, namely, the number of objects of that type, but you will always be able to get a larger number by going up to the next type. Therefore, there is no number so great but what you can get a greater number in a sufficiently high type. There you have the two sides of the argument: the one side when the type is given, the other side when the type is not given.

I have been talking, for brevity's sake, as if there really were all these different sorts of things. Of course, that is nonsense. There are particulars, but when one comes on to classes, and classes of classes, and classes of classes of classes, one is talking of logical fictions. When I say there are no such things, that again is not correct. It is not significant to say "There are such things" in the same sense of the words "there are" in which you can say "There are particulars." If I say "There are particulars" and "There are classes," the two phrases "there are" will have to have different meanings in those two propositions, and if they have suitable different meanings, both propositions may be true. If, on the other hand, the words "there are" are used in the same sense in both, then one at least of those statements must be nonsense, not false but nonsense. The question then arises, what is the sense in which one can say "There are classes," or in other words, what do you mean by a statement in which a class appears to come in? First of all, what are the sort of things you would like to say about classes? They are just the same as the sort of things you want to say about propositional functions. You want to say of a propositional function that it is sometimes true. That is the same thing as saying of a class that it has members. You want to say that it is true for exactly 100 values of the variables. That is the same as saying of a class that it has a hundred members. All the things you want to say about classes are the same as the things you want to say about propositional func-

tions excepting for accidental and irrelevant linguistic forms, with, however, a certain proviso which must now be explained.

Take, e. g., two propositional functions such as "x is a man," "x is a featherless biped." Those two are formally equivalent, i. e., when one is true so is the other, and *vice versa*. Some of the things that you can say about a propositional function will not necessarily remain true if you substitute another formally equivalent propositional function in its place. For instance, the propositional function "x is a man" is one which has to do with the concept of humanity. That will not be true of "x is a featherless biped." Or if you say, "So-and-so asserts that such-and-such is a man" the propositional function "x is a man" comes in there, but "x is a featherless biped" does not. There are a certain number of things which you can say about a propositional function which would be not true if you substitute another formally equivalent propositional function. On the other hand, any statement about a propositional function which will remain true or remain false, as the case may be, when you substitute for it another formally equivalent propositional function, may be regarded as being about the class which is associated with the propositional function. I want you to take the words *may be regarded* strictly. I am using them instead of *is*, because *is* would be untrue. "Extensional" statements about functions are those that remain true when you substitute any other formally equivalent function, and these are the ones that may be regarded as being about the class. If you have any statement about a function which is not extensional, you can always derive from it a somewhat similar statement which is extensional, viz., there is a function formally equivalent to the one in question about which the statement in question is true. This statement, which is manufactured out of the one you started with, will be ex-

tensional. It will always be equally true or equally false of any two formally equivalent functions, and this derived extensional statement may be regarded as being the corresponding statement about the associated class. So, when I say that "The class of men has so-and-so many members," that is to say "There are so-and-so many men in the world," that will be derived from the statement that "x is human" is satisfied by so-and-so many values of x, and in order to get it into the extensional form, one will put it in this way, that "There is a function formally equivalent to 'x is human,' which is true for so-and-so many values of x." That I should define as what I mean by saying "The class of men has so-and-so many members." In that way you find that all the formal properties that you desire of classes, all their formal uses in mathematics, can be obtained without supposing for a moment that there are such things as classes, without supposing, that is to say, that a proposition in which symbolically a class occurs, does in fact contain a constituent corresponding to that symbol, and when rightly analyzed that symbol will disappear, in the same sort of way as descriptions disappear when the propositions are rightly analyzed in which they occur.

There are certain difficulties in the more usual view of classes, in addition to those we have already mentioned, that are solved by our theory. One of these concerns the null-class, i. e., the class consisting of no members, which is difficult to deal with on a purely extensional basis. Another is concerned with unit-classes. With the ordinary view of classes you would say that a class that has only one member was the same as that one member. That will land you in terrible difficulties, because in that case that one member is a member of that class, namely, itself. Take, e. g., the class of "Lecture audiences in Gordon Square."[2] That is obviously a class of classes, and probably it is a

[2] [These lectures were given in Gordon Square, London.—Ed.]

class that has only one member, and that one member itself (so far) has more than one member. Therefore if you were to identify the class of lecture audiences in Gordon Square with the only lecture audience that there is in Gordon Square, you would have to say both that it has one member and that it has twenty members, and you will be landed in contradictions, because this audience has more than one member, but the class of audiences in Gordon Square has only one member. Generally speaking, if you have any collection of many objects forming a class, you can make a class of which that class is the only member, and the class of which that class is the only member will have only one member, though this only member will have many members. This is one reason why you must distinguish a unit-class from its only member. Another is that, if you do not, you will find that the class is a member of itself, which is objectionable, as we saw earlier in this lecture. I have omitted a subtlety connected with the fact that two formally equivalent functions may be of different types. For the way of treating this point, see *Principia Mathematica,* p. 20, and Introduction, Chap. III.

I have not said quite all that I ought to have said on this subject. I meant to have gone a little more into the theory of types. The theory of types is really a theory of symbols, not of things. In a proper logical language it would be perfectly obvious. The trouble that there is arises from our inveterate habit of trying to name what cannot be named. If we had a proper logical language, we should not be tempted to do that. Strictly speaking, only particulars can be named. In that sense in which there are particulars, you cannot say either truly or falsely that there is anything else. The word "there is" is a word having "systematic ambiguity," i. e., having a strictly infinite number of different meanings which it is important to distinguish.

DISCUSSION.

.....Could you lump all those classes, and classes of classes, and so on, together?

Mr. Russell: All are fictions, but they are different fictions in each case. When you say "There are classes of particulars," the statement "there are" wants expanding and explaining away, and when you have put down what you really do mean, or ought to mean, you will find that it is something quite different from what you thought. That process of expanding and writing down fully what you mean, will be different if you go on to "There are classes of classes of particulars." There are infinite numbers of meaning to "there are." The first only is fundamental, so far as the hierarchy of classes is concerned.

.....I was wondering whether it was rather analogous to spaces, where the first three dimensions are actual, and the higher ones are merely symbolic. I see there is a difference, there are higher dimensions, but you can lump those together.

Mr. Russell: There is only one fundamental one, which is the first one, the one about particulars, but when you have gone to classes, you have traveled already just as much away from what there is as if you have gone to classes of classes. There are no classes really in the physical world. The particulars are there, but not classes. If you say "There is a universe" that meaning of "there is" will be quite different from the meaning in which you say "There is a particular," which means that "the propositional function 'x is a particular' is sometimes true."

All those statements are about symbols. They are never about the things themselves, and they have to do with "types." This is really important and I ought not to have forgotten to say it, that the relation of the symbol to what it means is different in different types. I am not now talking about this hierarchy of classes and so on, but the relation of a predicate to what it means is different to the relation of a name to what it means. There is not one single concept of "meaning" as one ordinarily thinks there is, so that you can say in a uniform sense "All symbols have meaning," but there

are infinite numbers of different ways of meaning, i. e., different sorts of relation of the symbol to the symbolized, which are absolutely distinct. The relation, e. g., of a proposition to a fact, is quite different from the relation of a name to a particular, as you can see from the fact that there are two propositions always related to one given fact, and that is not so with names. That shows you that the relation that the proposition has to the fact is quite different from the relation of a name to a particular. You must not suppose that there is, over and above that, another way in which you could get at facts by naming them. You can always only get at the thing you are aiming at by the proper sort of symbol, which approaches it in the appropriate way. That is the real philosophical truth that is at the bottom of all this theory of types.

VIII. EXCURSUS INTO METAPHYSICS: WHAT THERE IS.

I come now to the last lecture of this course, and I propose briefly to point to a few of the morals that are to be gathered from what has gone before, in the way of suggesting the bearing of the doctrines that I have been advocating upon various problems of metaphysics. I have dwelt hitherto upon what one may call philosophical grammar, and I am afraid I have had to take you through a good many very dry and dusty regions in the course of that investigation, but I think the importance of philosophical grammar is very much greater than it is generally thought to be. I think that practically all traditional metaphysics is filled with mistakes due to bad grammar, and that almost all the traditional problems of metaphysics and traditional results—supposed results—of metaphysics are due to a failure to make the kind of distinctions in what we may call philosophical grammar with which we have been concerned in these previous lectures.

Take, as a very simple example, the philosophy of arithmetic. If you think that 1, 2, 3, and 4, and the rest of the numbers, are in any sense entities, if you think that there

are objects, having those names, in the realm of being, you have at once a very considerable apparatus for your metaphysics to deal with, and you have offered to you a certain kind of analysis of arithmetical propositions. When you say, e. g., that 2 and 2 are 4, you suppose in that case that you are making a proposition of which the number 2 and the number 4 are constituents, and that has all sorts of consequences, all sorts of bearings upon your general metaphysical outlook. If there has been any truth in the doctrines that we have been considering, all numbers are what I call logical fictions. Numbers are classes of classes, and classes are logical fictions, so that numbers are, as it were, fictions at two removes, fictions of fictions. Therefore you do not have, as part of the ultimate constituents of your world, these queer entities that you are inclined to call numbers. The same applies in many other directions.

One purpose that has run through all that I have said, has been the justification of analysis, i. e., the justification of logical atomism, of the view that you can get down in theory, if not in practice, to ultimate simples, out of which the world is built, and that those simples have a kind of reality not belonging to anything else. Simples, as I tried to explain, are of an infinite number of sorts. There are particulars and qualities and relations of various orders, a whole hierarchy of different sorts of simples, but all of them, if we were right, have in their various ways some kind of reality that does not belong to anything else. The only other sort of object you come across in the world is what we call *facts,* and facts are the sort of things that are asserted or denied by propositions, and are not properly entities at all in the same sense in which their constituents are. That is shown in the fact that you cannot name them. You can only deny, or assert, or consider them, but you cannot name them because they are not

there to be named, although in another sense it is true that you cannot know the world unless you know the facts that make up the truths of the world; but the knowing of facts is a different sort of thing from the knowing of simples.

Another purpose which runs through all that I have been saying is the purpose embodied in the maxim called Occam's Razor. That maxim comes in, in practice, in this way: take some science, say physics. You have there a given body of doctrine, a set of propositions expressed in symbols—I am including words among symbols—and you think that you have reason to believe that on the whole those propositions, rightly interpreted, are fairly true, but you do not know what is the actual meaning of the symbols that you are using. The meaning they have *in use* would have to be explained in some pragmatic way: they have a certain kind of practical or emotional significance to you which is a datum, but the logical significance is not a datum, but a thing to be sought, and you go through, if you are analyzing a science like physics, these propositions with a view to finding out what is the smallest empirical apparatus—or the smallest apparatus, not necessarily wholly empirical—out of which you can build up these propositions. What is the smallest number of simple undefined things at the start, and the smallest number of undemonstrated premises, out of which you can define the things that need to be defined and prove the things that need to be proved? That problem, in any case that you like to take, is by no means a simple one, but on the contrary an extremely difficult one. It is one which requires a very great amount of logical technique; and the sort of thing that I have been talking about in these lectures is the preliminaries and first steps in that logical technique. You cannot possibly get at the solution of such a problem as I am talking about if you go at it in a straightforward

fashion with just the ordinary acumen that one accumulates in the course of reading or in the study of traditional philosophy. You do need this apparatus of symbolical logic that I have been talking about. (The description of the subject as symbolical logic is an inadequate one. I should like to describe it simply as logic, on the ground that nothing else really is logic, but that would sound so arrogant that I hesitate to do so.)

Let us consider further the example of physics for a moment. You find, if you read the works of physicists, that they reduce matter down to certain elements—atoms, ions, corpuscles, or what not. But in any case the sort of thing that you are aiming at in the physical analysis of matter is to get down to very little bits of matter that still are just like matter in the fact that they persist through time, and that they travel about in space. They have in fact all the ordinary every-day properties of physical matter, not the matter that one has in ordinary life—they do not taste or smell or appear to the naked eye—but they have the properties that you very soon get to when you travel toward physics from ordinary life. Things of that sort, I say, are not the ultimate constituents of matter in any metaphysical sense. Those things are all of them, as I think a very little reflection shows, logical fictions in the sense that I was speaking of. At least, when I say they are, I speak somewhat too dogmatically. It is possible that there may be all these things that the physicist talks about in actual reality, but it is impossible that we should ever have any reason whatsoever for supposing that there are. That is the situation that you arrive at generally in such analyses. You find that a certain thing which has been set up as a metaphysical entity can either be assumed dogmatically to be real, and then you will have no possible argument either for its reality or against its reality; or, instead of doing that, you can construct a logical fiction

having the same formal properties, or rather having formally analogous formal properties to those of the supposed metaphysical entity and itself composed of empirically given things, and that logical fiction can be substituted for your supposed metaphysical entity and will fulfil all the scientific purposes that anybody can desire. With atoms and the rest it is so, with all the metaphysical entities whether of science or of metaphysics. By metaphysical entities I mean those things which are supposed to be part of the ultimate constituents of the world, but not to be the kind of thing that is ever empirically given,—I do not say merely not being itself empirically given, but not being the *kind* of thing that is empirically given. In the case of matter, you can start from what is empirically given, what one sees and hears and smells and so forth, all the ordinary data of sense, or you can start with some definite ordinary object, say this desk, and you can ask yourselves, "What do I mean by saying that this desk that I am looking at now is the same as the one I was looking at a week ago?" The first simple ordinary answer would be that it *is* the same desk, it is actually identical, there is a perfect identity of substance, or whatever you like to call it. But when that apparently simple answer is suggested, it is important to observe that you cannot have an empirical reason for such a view as that, and if you hold it, you hold it simply because you like it and for no other reason whatever. All that you really know is such facts as that what you see now, when you look at the desk, bears a very close similarity to what you saw a week ago when you looked at it. Rather more than that one fact of similarity I admit you know, or you may know. You might have paid some one to watch the desk continuously throughout the week, and might then have discovered that it was presenting appearances of the same sort all through that period, assuming that the light was kept on all through the night. In that way you could

have established continuity. You have not in fact done so. You do not in fact know that that desk has gone on looking the same all the time, but we will assume that. Now the essential point is this: What is the empirical reason that makes you call a number of appearances, appearances of the same desk? What makes you say on successive occasions, I am seeing the same desk? The first thing to notice is this, that it does not matter what is the answer, so long as you have realized that the answer consists in something empirical and not in a recognized metaphysical identity of substance. There is something given in experience which makes you call it the same desk, and having once grasped that fact, you can go on and say, it is that something (whatever it is) that makes you call it the same desk which shall be *defined* as *constituting* it the same desk, and there shall be no assumption of a metaphysical substance which is identical throughout. It is a little easier to the untrained mind to conceive of an identity than it is to conceive of a system of correlated particulars, hung one to another by relations of similarity and continuous change and so on. That idea is apparently more complicated, but that is what is empirically given in the real world, and substance, in the sense of something which is continuously identical in the same desk, is not given to you. Therefore in all cases where you seem to have a continuous entity persisting through changes, what you have to do is to ask yourself what makes you consider the successive appearances as belonging to one thing. When you have found out what makes you take the view that they belong to the same thing, you will then see that that which has made you say so, is all that is *certainly* there in the way of unity. Anything that there may be over and above that, I shall recognize as something I cannot know. What I can know is that there are a certain series of appearances linked together, and the series of those appearances I shall define

as being a desk. In that way the desk is reduced to being a logical fiction, because a series is a logical fiction. In that way all the ordinary objects of daily life are extruded from the world of what there is, and in their place as what there is you find a number of passing particulars of the kind that one is immediately conscious of in sense. I want to make clear that I am not *denying* the existence of anything; I am only refusing to affirm it. I refuse to affirm the existence of anything for which there is no evidence, but I equally refuse to deny the existence of anything against which there is no evidence. Therefore I neither affirm nor deny it, but merely say, that is not in the realm of the knowable and is certainly not a part of physics; and physics, if it is to be interpreted, must be interpreted in terms of the sort of thing that can be empirical. If your atom is going to serve purposes in physics, as it undoubtedly does, your atom has got to turn out to be a construction, and your atom will in fact turn out to be a series of classes of particulars. The same process which one applies to physics, one will also apply elsewhere. The application to physics I explained briefly in my book on the *External World,* Chapters III and IV (Open Court Publishing Co., 1914).

I have talked so far about the unreality of the things we think real. I want to speak with equal emphasis about the reality of things we think unreal, such as phantoms and hallucinations. Phantoms and hallucinations, considered in themselves, are, as I explained in the preceding lectures, on exactly the same level as ordinary sense-data. They differ from ordinary sense-data only in the fact that they do not have the usual correlations with other things. In themselves they have the same reality as ordinary sense-data. They have the most complete and absolute and perfect reality that anything can have. They are part of the ultimate constituents of the world, just as the fleeting

sense-data are. Speaking of the fleeting sense-data, I think it is very important to remove out of one's instincts any disposition to believe that the real is the permanent. There has been a metaphysical prejudice always that if a thing is really real, it has to last either forever or for a fairly decent length of time. That is to my mind an entire mistake. The things that are really real last a very short time. Again I am not denying that there *may* be things that last forever, or for thousands of years; I only say that those are not within our experience, and that the real things that we know by experience last for a very short time, one tenth or half a second, or whatever it may be. Phantoms and hallucinations are among those, among the ultimate constituents of the world. The things that we call real, like tables and chairs, are systems, series of classes of particulars, and the particulars are the real things, the particulars being sense-data when they happen to be given to you. A table or chair will be a series of classes of particulars, and therefore a logical fiction. Those particulars will be on the same level of reality as a hallucination or a phantom. I ought to explain in what sense a chair is a series of classes. A chair presents at each moment a number of different appearances. All the appearances that it is presenting at a given moment make up a certain class. All those sets of appearances vary from time to time. If I take a chair and smash it, it will present a whole set of different appearances from what it did before, and without going as far as that, it will always be changing as the light changes, and so on. So you get a series in time of different sets of appearances, and that is what I mean by saying that a chair is a series of classes. That explanation is too crude, but I leave out the niceties, as that is not the actual topic I am dealing with. Now each single particular which is part of this whole system is linked up with the others in the system. Supposing, e. g., I take as my par-

ticular the appearance which that chair is presenting to me at this moment. That is linked up first of all with the appearance which the same chair is presenting to any one of you at the same moment, and with the appearance which it is going to present to me at later moments. There you get at once two journeys that you can take away from that particular, and that particular will be correlated in certain definite ways with the other particulars which also belong to that chair. That is what you mean by saying—or what you ought to mean by saying—that what I see before me is a real thing as opposed to a phantom. It means that it has a whole set of correlations of different kinds. It means that that particular, which is the appearance of the chair to me at this moment, is not isolated but is connected in a certain well-known familiar fashion with others, in the sort of way that makes it answer one's expectations. And so, when you go and buy a chair, you buy not only the appearance which it presents to you at that moment, but also those other appearances that it is going to present when it gets home. If it were a phantom chair, it would not present any appearances when it got home, and would not be the sort of thing you would want to buy. The sort one calls real is one of a whole correlated system, whereas the sort you call hallucinations are not. The respectable particulars in the world are all of them linked up with other particulars in respectable, conventional ways. Then sometimes you get a wild particular, like a merely visual chair that you cannot sit on, and say it is a phantom, a hallucination, you exhaust all the vocabulary of abuse upon it. That is what one means by calling it unreal, because "unreal" applied in that way is a term of abuse and never would be applied to a thing that *was* unreal because you would not be so angry with it.

I will pass on to some other illustrations. Take a person. What is it that makes you say, when you meet

your friend Jones, "Why, this is Jones"? It is clearly not the persistence of a metaphysical entity inside Jones somewhere, because even if there be such an entity, it certainly is not what you see when you see Jones coming along the street; it certainly is something that you are not acquainted with, not an empirical datum. Therefore plainly there is something in the empirical appearances which he presents to you, something in their relations one to another, which enables you to collect all these together and say, "These are what I call the appearances of one person," and that something that makes you collect them together is not the persistence of a metaphysical subject, because that, whether there be such a persistent subject or not, is certainly not a datum, and that which makes you say "Why, it is Jones" is a datum. Therefore Jones is not constituted as he is known by a sort of pin-point ego that is underlying his appearances, and you have got to find some correlations among the appearances which are of the sort that make you put all those appearances together and say, they are the appearances of one person. Those are different when it is other people and when it is yourself. When it is yourself, you have more to go by. You have not only what you look like, you have also your thoughts and memories and all your organic sensations, so that you have a much richer material and are therefore much less likely to be mistaken as to your own identity than as to some one else's. It happens, of course, that there are mistakes even as to one's own identity, in cases of multiple personality and so forth, but as a rule you will know that it is you because you have more to go by than other people have, and you would know it is you, not by a consciousness of the ego at all but by all sorts of things, by memory, by the way you feel and the way you look and a host of things. But all those are empirical data, and those enable you to say that the person to whom something happened yesterday

was yourself. So you can collect a whole set of experiences into one string as all belonging to you, and similarly other people's experiences can be collected together as all belonging to them by relations that actually are observable and without assuming the existence of the persistent ego. It does not matter in the least to what we are concerned with, what exactly is the given empirical relation between two experiences that makes us say, "These are two experiences of the same person." It does not matter precisely what that relation is, because the logical formula for the construction of the person is the same whatever that relation may be, and because the mere fact that you can know that two experiences belong to the same person proves that there is such an empirical relation to be ascertained by analysis. Let us call the relation R. We shall say that when two experiences have to each other the relation R, then they are said to be experiences of the same person. That is a definition of what I mean by "experiences of the same person." We proceed here just in the same way as when we are defining numbers. We first define what is meant by saying that two classes "have the same number," and then define what a number is. The person who has a given experience x will be the class of all those experiences which are "experiences of the same person" as the one who experiences x. You can say that two events are co-personal when there is between them a certain relation R, namely that relation which makes us say that they are experiences of the same person. You can define the person who has a certain experience as being those experiences that are co-personal with that experience, and it will be better perhaps to take them as a series than as a class, because you want to know which is the beginning of a man's life and which is the end. Therefore we shall say that a person is a certain series of experiences. We shall not deny that there may be a metaphysical ego. We shall merely say

that it is a question that does not concern us in any way, because it is a matter about which we know nothing and can know nothing, and therefore it obviously cannot be a thing that comes into science in any way. What we know is this string of experiences that makes up a person, and that is put together by means of certain empirically given relations, such, e. g., as memory.

I will take another illustration, a kind of problem that our method is useful in helping to deal with. You all know the American theory of neutral monism, which derives really from William James and is also suggested in the work of Mach, but in a rather less developed form. The theory of neutral monism maintains that the distinction between the mental and the physical is entirely an affair of arrangement, that the actual material arranged is exactly the same in the case of the mental as it is in the case of the physical, but they differ merely in the fact that when you take a thing as belonging in the same context with certain other things, it will belong to psychology, while when you take it in a certain other context with other things, it will belong to physics, and the difference is as to what you consider to be its context, just the same sort of difference as there is between arranging the people in London alphabetically or geographically. So, according to William James, the actual material of the world can be arranged in two different ways, one of which gives you physics and the other psychology. It is just like rows or columns: in an arrangement of rows and columns, you can take an item as either a member of a certain row or a member of a certain column; the item is the same in the two cases, but its context is different.

If you will allow me a little undue simplicity I can go on to say rather more about neutral monism, but you must understand that I am talking more simply than I ought to do because there is not time to put in all the shadings and

qualifications. I was talking a moment ago about the appearances that a chair presents. If we take any one of these chairs, we can all look at it, and it presents a different appearance to each of us. Taken all together, taking all the different appearances that that chair is presenting to all of us at this moment, you get something that belongs to physics. So that, if one takes sense-data and arranges together all those sense-data that appear to different people at a given moment and are such as we should ordinarily say are appearances of the same physical object, then that class of sense-data will give you something that belongs to physics, namely, the chair at this moment. On the other hand, if instead of taking all the appearances that that chair presents to all of us at this moment, I take all the appearances that the different chairs in this room present to me at this moment, I get quite another group of particulars. All the different appearances that different chairs present to me now will give you something belonging to psychology, because that will give you my experiences at the present moment. Broadly speaking, according to what one may take as an expansion of William James, that should be the definition of the difference between physics and psychology.

We commonly assume that there is a phenomenon which we call seeing the chair, but what I call my seeing the chair according to neutral monism is merely the existence of a certain particular, namely the particular which is the sense-datum of that chair at that moment. And I and the chair are both logical fictions, both being in fact a series of classes of particulars, of which one will be that particular which we call my seeing the chair. That actual appearance that the chair is presenting to me now is a member of me and a member of the chair, I and the chair being logical fictions. That will be at any rate a view that you can consider if you are engaged in vindicating neutral monism.

There is no simple entity that you can point to and say: this entity is physical and not mental. According to William James and neutral monists that will not be the case with any simple entity that you may take. Any such entity will be a member of physical series and a member of mental series. Now I want to say that if you wish to test such a theory as that of neutral monism, if you wish to discover whether it is true or false, you cannot hope to get any distance with your problem unless you have at your fingers' end the theory of logic that I have been talking of. You never can tell otherwise what can be done with a given material, whether you can concoct out of a given material the sort of logical fictions that will have the properties you want in psychology and in physics. That sort of thing is by no means easy to decide. You can only decide it if you really have a very considerable technical facility in these matters. Having said that, I ought to proceed to tell you that I have discovered whether neutral monism is true or not, because otherwise you may not believe that logic is any use in the matter. But I do not profess to know whether it is true or not. I feel more and more inclined to think that it may be true. I feel more and more that the difficulties that occur in regard to it are all of the sort that may be solved by ingenuity. But nevertheless there *are* a number of difficulties; there are a number of problems, some of which I have spoken about in the course of these lectures. One is the question of belief and the other sorts of facts involving two verbs. If there are such facts as this, that, I think, may make neutral monism rather difficult, but as I was pointing out, there is the theory that one calls behaviorism, which belongs logically with neutral monism, and that theory would altogether dispense with those facts containing two verbs, and would therefore dispose of that argument against neutral monism. There is, on the other hand, the argument from emphatic par-

ticulars, such as "this" and "now" and "here" and such words as that, which are not very easy to reconcile, to my mind, with the view which does not distinguish between a particular and experiencing that particular. But the argument about emphatic particulars is so delicate and so subtle that I cannot feel quite sure whether it is a valid one or not, and I think the longer one pursues philosophy, the more conscious one becomes how extremely often one has been taken in by fallacies, and the less willing one is to be quite sure that an argument is valid if there is anything about it that is at all subtle or elusive, at all difficult to grasp. That makes me a little cautious and doubtful about all these arguments, and therefore although I am quite sure that the question of the truth or falsehood of neutral monism is not to be solved except by these means, yet I do not profess to know whether neutral monism is true or is not. I am not without hopes of finding out in the course of time, but I do not profess to know yet.

As I said earlier in this lecture, one thing that our technique does, is to give us a means of constructing a given body of symbolic propositions with the minimum of apparatus, and every diminution in apparatus diminishes the risk of error. Suppose, e. g., that you have constructed your physics with a certain number of entities and a certain number of premises; suppose you discover that by a little ingenuity you can dispense with half of those entities and half of those premises, you clearly have diminished the risk of error, because if you had before 10 entities and 10 premises, then the 5 you have now would be all right, but it is not true conversely that if the 5 you have now are all right, the 10 must have been. Therefore you diminish the risk of error with every diminution of entities and premises. When I spoke about the desk and said I was not going to assume the existence of a persistent substance underlying its appearances, it is an example of the case in

point. You have anyhow the successive appearances, and if you can get on without assuming the metaphysical and constant desk, you have a smaller risk of error than you had before. You would not necessarily have a smaller risk of error if you were tied down to *denying* the metaphysical desk. That is the advantage of Occam's Razor, that it diminishes your risk of error. Considered in that way you may say that the whole of our problem belongs rather to science than to philosophy. I think perhaps that is true, but I believe the only difference between science and philosophy is, that science is what you more or less know and philosophy is what you do not know. Philosophy is that part of science which at present people choose to have opinions about, but which they have no knowledge about. Therefore every advance in knowledge robs philosophy of some problems which formerly it had, and if there is any truth, if there is any value in the kind of procedure of mathematical logic, it will follow that a number of problems which had belonged to philosophy will have ceased to belong to philosophy and will belong to science. And of course the moment they become soluble, they become to a large class of philosophical minds uninteresting, because to many of the people who like philosophy, the charm of it consists in the speculative freedom, in the fact that you can play with hypotheses. You can think out this or that which *may* be true, which is a very valuable exercise until you discover what *is* true; but when you discover what is true the whole fruitful play of fancy in that region is curtailed, and you will abandon that region and pass on. Just as there are families in America who from the time of the Pilgrim Fathers onward had always migrated westward, toward the backwoods, because they did not like civilized life, so the philosopher has an adventurous disposition and likes to dwell in the region where there are still uncertainties. It is true that the transferring of a

region from philosophy into science will make it distasteful to a very important and useful type of mind. I think that is true of a good deal of the applications of mathematical logic in the directions that I have been indicating. It makes it dry, precise, methodical, and in that way robs it of a certain quality that it had when you could play with it more freely. I do not feel that it is my place to apologize for that, because if it is true, it is true. If it is not true, of course, I do owe you an apology; but if it is, it is not my fault, and therefore I do not feel I owe any apology for any sort of dryness or dulness in the world. I would say this too, that for those who have any taste for mathematics, for those who like symbolic constructions, that sort of world is a very delightful one, and if you do not find it otherwise attractive, all that is necessary to do is to acquire a taste for mathematics, and then you will have a very agreeable world, and with that conclusion I will bring this course of lectures to an end.

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NOTES ON MAHAYANA BUDDHISM.

III. MAHAYANA BUDDHISM AS PSYCHOLOGY.

The Existence of the Soul.—We now come to a discussion of what many consider the most difficult part of Buddhism, and which is, beyond doubt, the point most misunderstood by all Occidental students of that faith, whether Hinayana or Mahayana—the question as to the nature of the soul. It is often said that Buddhism denies the existence of the soul, but this is a gross misrepresentation. It does, indeed, reject the idea of the soul being an absolute thing, a thing in itself, but to Buddhism the existence of the soul, though only as a compound, is a thing indisputable and not requiring proof.

In order to understand fully the teachings of the Buddha concerning the nature and fate of the soul, it is first of all necessary to know something of the psychological opinions which were current in India at the time of his life. According to these, the "soul" consisted of a collection of "bodies"—the material, the emotional, mental, spiritual, etc.—which are constantly changing, but which are united by a permanent and unchanging *atman* or ego-entity.[16] Now Hinayana affirms, as also does Mahayana in its negative phase, the same idea of the soul, except that it denies such a thing as an atman which binds the various "bodies"

[16] It is to be noted that there is a sharp difference between Christianity and Hinduism in regard to the soul, though both accept its absolute existence. In a general way, however, it may be said that the "soul" of Paul corresponds to the various bodies taken together and the "spirit" to the atman.

together. In other words, it teaches that the soul consists of five *skandahs* (mental qualities) which have been gathered together as a result of the actions of a former group of skandahs, and that the only link between them is the sequence of cause and effect.

The Atman in Mahayana.—Mahayana in its positive phase, however, in contradistinction to Hinayana, differs a little from this. The atman, according to Brahmanism, is a spark of the divine, and Mahayana, in the sense that each individual is a manifestation of the divine, teaches that there is an atman. It holds, however, that there is only one atman, the Bhutatathata of which we are all aspects, and that the divine spirit within A is not different from the divine spirit within B. In other words, the distinction between Mahayana and the popular Hindu theory of the atman, is that the latter teaches that there was originally one great ocean of life (Parabrahm) from which a portion has been taken and put into each individual so that the divine drop within each person is distinct and separate from that of every other person, and all the drops, in turn, are separate and distinct from the great ocean; while Mahayana, on the other hand, holds that there is only the great ocean (Bhutatathata) and that although there are many waves (personalities) the real essence of each wave (the water) remains at one with the other waves.[17]

In short, the Mahayanist asserts that the great divine ocean, if it be truly divine, must be omnipresent and all-pervading, and that consequently we must all exist in it. Now if we live in the divine ocean, and the drop within each one of us is of exactly the same nature as the ocean itself (as Hinduism teaches) it follows necessarily that the drop must be one with the universal ocean and indistin-

[17] As a matter of fact, several of the Vedantin schools of India hold to the Mahayana idea on this point, thus making Mahayana and Vedantism almost identical, except of course for verbal and minor differences, and they may be regarded as different sects of the same religion rather than as separate and distinct faiths.

guishable from it, just as a drop is one with and indistinguishable in the material ocean.[18] Certainly, however false or true either one of the two theories may be, that of Mahayana is the more logical.

The Persistence of the Personality.—But, it may be objected, however fine this may be theoretically, what of its practical application? What of the survival of the personality? Does Buddhism deny or affirm that the individuality, the I and the you, continue to exist after death? Buddhism in answer to this question, affirms, characteristically, that it both does and does not, or, in other words, that it persists in one sense and does not persist in another sense. This, of course, at first sight seems foolish and illogical, but Buddhism will point out the human body in defense—in one sense it persists from year to year while in another sense it does not, for we all know that the particles of which the body is composed are constantly changing, and physiology even tells us that in seven years it is an entirely new body, having not a single material thing in it which it had seven years previously, and yet do we not speak at the same time of the same body persisting from babyhood until death? In the same way the Buddhist will speak of the persistence and non-persistence of the soul. The point which we have noted—namely that according to Buddhism the soul is a compound thing and not a thing in itself—must not be forgotten. Excepting the four limbs, the trunk and the head, where is the body? In like manner according to Buddhism, excepting the five skandahs, where is the individual soul? And as we know the body, i. e., matter which composes the limbs, trunk, and head, is constantly changing, therefore says the Buddhist, why not the soul?

[18] Except that the material ocean is composed of atoms and that, since space must exist between atoms, the water does not really cover all the space it seemingly occupies. The divine ocean, however, must be non-atomical since it is absolutely omnipresent, and consequently no space however minute can divide one portion from another.

The Nature of the Individuality.—This question as to the persistence and the non-persistence of the personality may be likened to the wheel of a cart, a simile which has often been used by Buddhist writers since it gives an excellent idea of *non-atman*—for except for the hub, spokes, and rim, what is there? Now supposing that one of the spokes on the wheel were to wear out and were replaced, and then another, and so on, and finally the hub and the rim; there would be nothing left of the original wheel, and yet we would be justified in speaking about it, in one sense, as the same wheel. We would be justified in saying that the personality (that is the actual materials) of the wheel is constantly changing and is perishable, yet we may also say that the individuality (that is the wheel taken as a whole) persists, though by individuality we do not mean a certain concrete thing, such as the hub, or one of the spokes, or the rim. This remains changeless in the midst of change. In like manner the Buddhist declares that while the individuality is ever changing, the individuality continues, though by individuality is meant not an imperishable ego-entity, but merely the sequence of cause and effect.

The Law of Karma.—Buddhism is essentially scientific inasmuch as it lays especial stress upon the law of cause and effect, or in Buddhist phraseology, *karma,* that which Saint Paul spoke of when he said, "As ye sow, so shall ye reap." All philosophies have in a general way accepted it, but Buddhism has given it especial emphasis. This fact, considering that Buddhism was formulated centuries before the time of Christ, long before the discovery of the laws of the conservation of matter, and of energy, is little less than remarkable.

Karma to Buddhism, however, has a little different significance than to any other religious or philosophical system, on account of its denial of the ego-entity. Karma, or the idea of retribution, in other systems means that what

a personality sows it reaps, while in Buddhism it signifies the method of the formation of a new personality as well. Here again, when the idea thus seems difficult to understand, physiology comes to the rescue. Take the body for example: in one sense it may be said that the body reaps what it sows—thus when it has been strongly under the influence of liquor it has a headache—yet to speak more exactly, an intoxicated body causes the formation of an ill body, for if the body is not the same for two consecutive moments, it follows that in one sense the body which was intoxicated is a different one from that body which suffers the consequences.

In like manner, the karma of our present personality will, according to Buddhism, result in the formation of a new personality, though, as we have already seen, the two personalities are in one sense identical. Thus, for example, we are told that the deeds and thoughts of a vicious man will, upon his death, result in the formation of a new personality in one of the hells, just as those of the virtuous man in one of the heavens.

Reincarnation.—Buddhism of both branches, however, in addition to the idea of a rebirth in a heaven or hell, teaches that a reincarnation may also occur on earth. In other words, the karma or the fruits of our actions, whether good or bad, will result in the formation of a new personality in this world again. In this way, Buddhism accounts for the many seeming injustices which are everywhere apparent and which, in the West, have been the cause of so much questioning as to the wisdom and mercy of God, since, of course, Buddhism teaches that the state of the new personality will be conditioned by the acts of its predecessor.

I have used the words "new personality" and "predecessor," but, as has already been noted, it is in one way the same person who both sows and reaps. Certainly it is

far nearer the truth to say that they are absolutely identical than that they are absolutely different, just as it is far nearer the truth to say that the man who committed a murder and the man who is justly hanged for it is the same than that they are two different individuals.

Buddhism, however, since it rejects the idea of a mysterious ego-entity which departs from one body and goes into another, prefers to speak not of transmigration, which gives too atmanistic a conception, but of reincarnation or rebirth. What is it that is reborn? As we have already seen, it is the karma of the dead man, the fruit of his deeds, and in that sense the man himself; but it is also more than that, since Buddhism teaches that a man's desires or wishes are reincarnated. What are we but a bundle of sensations and desires? Yet Buddhism assures us that these wishes live on in the new personality long after the old personality has decayed.

The Nature of the Rebirth.—While the Buddha affirmed the truth of the idea of reincarnation in no uncertain terms, many of the questions as to the exact method by which it occurred were left unanswered by him. In popular Hinduism, the atman was supposed to leave the old body at death at the top of the head and enter into the body newly prepared for it, by the same means. In Buddhism with its denial of the atman, this was of course impossible, but the Buddha as far as we have any authentic record, does not seem to have taken the trouble to formulate any substitute hypothesis as to the nature of the link between the two personalities; nor has Mahayana, in spite of its development along other lines, ever apparently felt the need of dealing more in detail with the matter.

Then too, Buddhism, while closely linking the ideas of retribution and reincarnation, has never been very explicit as to just what does determine the future state of man, whether it is his deeds or his thoughts alone, or both. In

the different moral stories which the Buddha is supposed to have related concerning his own and others' past lives, the cause of the state of existence varies widely. Sometimes it would appear to be the net balance of the whole of a man's actions, sometimes a single exceptional thought or deed, or again it would be merely the result of a strong desire held when the person was dying. The reincarnation is not, according to popular Buddhism, limited to the human kingdom, it being possible for a man to be reborn as an animal, and *vice versa,* while, on the other hand, a man may rise and in his next existence become a god.

There is also some ambiguity as to whether the "soul" enters at once into a new physical body or remains in a disembodied form for some time. Some of the *Jataka* or rebirth tales would seem to imply the former, but modern Mahayana for the most part holds that the person remains some time in one of the heavens or hells before being reincarnated. This metempsychosis continues until Nirvana or Buddhahood is attained, when one at length secures freedom from the chain of birth and death.

IV. MAHAYANA BUDDHISM AS ETHICS.

Anatman and Morality.—It is often objected that the Buddhist conception of the soul is conducive to immortality. If people think, it is alleged, that if even in one sense it is another person who will reap the benefits of their good deeds, they will cease from striving, or that another person will be punished for their vices, they will give themselves over to licentiousness. The Buddhist will answer, however, that this conception is justified neither by logic nor by its results in the countries where the doctrine of *anatman* has been applied. Look at the people of China and Japan, and the other Buddhist nations, he will say, and whatever may be their individual moral failings, one must note that they go on striving in order that their next birth may be

a fortunate one. We find them in their old age rejoicing at the fact that they will soon be provided with a new body: we find them praying that they may be reborn in the Pure Land (*jodo*) that they may more quickly enter Nirvana. And this not because of the fact that among the lower classes the idea of anatman is not fully understood or believed, as has been sometimes objected, for I have found by personal observation that the peasants as well as the educated classes are acquainted in a general way with the doctrine and fully believe in it. And, certainly, even supposing that the objection were true, do we not find among the monks who assuredly comprehend and uphold it, a rigid morality, a stern austerity? It would be nearer the truth, the Buddhist will affirm, to say that in actual life the idea leads to asceticism rather than to self-indulgence.

Moreover, he will go further than this, for he will cite the behavior of people all the world over. The body that swallows the poison is not the same body (in one sense) that dies—but does that mean that a knowledge of and belief in this fact causes men to drink strychnine? We know that the man who eats injudiciously and the man who suffers the consequences are in one sense different, yet knowing this fact are we any less careful of our digestions? The person who carefully saves his money and the person who at the end of a lifetime has the fortune are as much different as Buddhism teaches that he will be in his next life, yet would it be fair, argues the Buddhist, to lay all extravagance to this fact? Why, therefore, he will argue, should we say that the doctrine of anatman leads to immorality?

The Threefold Path.—What, then, *is* the moral law of Buddhism? What is it that the Buddha instructed his disciples to do in order to acquire merit? Speaking on this subject, the Rt. Rev. Shaku Soyen, Head Abbot of the Zen sect and one of the leading lights of Buddhism, says,

"It (the moral law) is threefold, (1) to cease from doing evil; (2) to promote goodness; and (3) to enlighten the ignorant. In this conception, Buddhist ethics is the simplest thing in the world. In the latter part of the Tang dynasty in China there was a famous poet-statesman who is known in Japanese as Hak-rak-ten. He learned that there resided in his district a Buddhist monk greatly noted for his saintly life and scholarly learning, and he went to see him intending to discourse with him on some deep religious subjects. Ushered in to his presence, the poet inquired what the monk thought to be the most influential thing in Buddhism. The monk replied without hesitation that it was the teaching of all unenlightened beings, to cease from doing evil, to promote goodness, and to purify one's heart. Hak-rak-ten was nonplussed to receive such a commonplace from such a renowned personage professing the faith of Buddhism, for he had expected something highly speculative and metaphysical. . . . The poet therefore sharply retorted, "This is what every child of three summers is acquainted with. I, on the other hand, desire to know what is most abstruse, most essential and vital in Buddhism." The monk, however, coldly replied, "Every child of three summers may know what I said now, but even a silver-haired old man of eighty winters finds it difficult to put the Buddhist instructions into the practice of every-day life." And it is said that thereupon the poet-statesman reverentially bowed and went home full of thought.

The Eightfold Path.—As a matter of fact, however, Buddhist morality is by no means as simple as that, since we have elaborations of this threefold path in many shapes and guises. The most noted of these, because of the fact that it is supposed to have been preached by the Buddha himself, is the so-called Noble Eightfold Path, which we

have already seen to be one of the points common to Mahayana and Hinayana alike.

This is, as has been seen, (1) right belief or comprehension, (2) right aspiration, (3) right speech, (4) right deeds, (5) right mode of livelihood, (6) right effort or endeavor, (7) right mindfulness or self-discipline, and lastly (8) right rapture or ecstasy. This division of the moral code is simple (almost childish, one might say) yet covers practically all the ground. One or two things about it, however, require especial notice. To begin with, what in Christian terminology would be called *faith* is regarded as only one of eight necessary qualifications, amounting to much less than in Christian theology; yet, considering the influence it has upon one's character, the Buddha was certainly justified in putting it first and foremost in his list of the necessary virtues. The Buddha also evidently recognized the truth which was expressed by the Hebrew prophet who proclaimed that where there is no vision the people perish, when he placed "right aspiration" or the possession of high ideals as one of the most important of the moral qualifications. The only other point which requires any explanation is the seventh one, namely, that of self-discipline. It must not be supposed from this that Buddhism teaches asceticism or self-mortification, since we know that the Buddha made it one of his especial aims to denounce the absurd lengths to which the Hindus of his day had gone in this respect. In fact we find him proclaiming in his first sermon, "There are two extremes which he who has gone forth[19] ought not to follow—habitual devotion to the passions on the one hand, to the pleasures of sensual things, etc., and habitual devotion on the other hand to self-mortification, which is painful, ignoble, and unprofitable." And we find throughout his life, his

[19] This term signifies one who seeks after enlightenment, and is the word which is commonly used to express this idea.

use of the word "the Middle Path" (between the two extremes) to denote his teachings.

The Fetters of the Path.—In addition to the positive moral code which has been outlined, Buddhism has also a negative one, i. e., one which declares what things are evil and to be avoided. First and foremost there are the four intoxicants which hold back the seeker after enlightenment, namely, (1) mental infatuation arising from sensual pleasures, (2) that which arises from pride of life, (3) from ignorance, and lastly (4) from useless speculation over what is unsolvable.

Then there are the ten chief bonds, or lions in the Path: (1) belief in the atman, (2) doubt, (3) dependence upon works, (4) sensuality, (5) hatred, (6) desire for rebirth on earth, (7) or in heaven, (8) conceit, (9) self-righteousness, and last but not least (10) ignorance. These bonds may be classified into five groups consisting of: (1) the first three, which are supposed to be swept away upon "conversion" to Buddhism (if I may be permitted such a word); (2) next the fourth and the fifth, the so-called coarser bonds; (3) then the sixth and the seventh, the finer bonds, of desire; (4) then pride and self-righteousness against which the Christ so vigorously protested; and lastly (5) ignorance, the final fetter, which when cast aside gives perfect enlightenment to the seeker.

The Practical Side of Buddhist Morality.—Such may be said to be the theoretical side of Buddhist morality, the mental states to be cultivated and avoided, etc., but in addition to this, Buddhist morality has also its practical aspect. Speaking generally it may be said that this aspect has five chief features, namely, (1) non-sensuality, (2) sobriety, (3) harmlessness, (4) truthfulness, and (5) honesty. Of the first, of the necessity of leading a chaste life, we have already spoken, lust being one of the ten bonds above mentioned. The Buddha seems to have laid especial

emphasis on this point, and we find the Sutra of the Forty-Two Sections, one of the principal Mahayana Sutras, declaring as the words of the Buddha, that it were well that there is no other fetter as powerful as lust, since if there were, to follow the Noble Path would then be impossible. Concerning the second point, it is of interest to note that the Buddha was evidently several centuries ahead of his time as far as the drink problem is concerned, since we find him forbidding the partaking of all forms of alcoholic liquors. The third feature shows itself in an unwillingness to kill any creature, whether human or animal. In consequence of this we have two further main precepts of Buddhism, non-resistance and vegetarianism. The fourth and fifth precepts are, of course, common to all ethics as well as to all religions, and so require no further explanation or elucidation.

Non-Observance of the Moral Law.—As is the case with Christian morality in Christian countries, many points of the high and stern moral law of the Buddha are often relaxed in individual observance. One of the principal reasons for this may be found in the external organization of Buddhism, which, whatever its merits may be, has certainly its demerits in this respect. According to strict Buddhism only the monks are true Buddhists, the laymen, however pious, are mere half-hearted ones. The priests are not, as in Christian countries, the leaders and teachers of the people, but persons who draw aside from the world that they may live a more perfect life and win enlightenment and salvation,[20] while the laymen are looked upon as those who outside of the church support the monks in "acquiring merit." Such an order of things would necessarily lead to laxity among the laymen, and accordingly not only do we find them marrying, as is of course to be expected, but also

[20] Here again the Shin Shu differs from ordinary Buddhism, inasmuch as its priests go out into the world far more than do the priests of the other sects, and their duties approach very nearly those of the Christian clergyman.

drinking wine and eating meat. However, enough of the Buddhist spirit prevails to cause butchers and brewers to be regarded as among the basest members of society, and an unwillingness is manifest to kill any animal directly even though it will be eaten without compunction after it has been slaughtered by some one else.

Another and stronger reason for the weakening of these basic rules, however, is that Mahayana (not Hinayana) Buddhism has absorbed something of the spirit of the end justifying the means. Thus while the Buddha preached of the value of peace at all costs, we find Mahayana teaching that while war is undoubtedly bad, it certainly would be worse to see the good suffer, and so, unlike Hinayana which still proclaims the doctrine of absolute non-resistance,[21] we find the Mahayana priests not only not decrying war but actually encouraging it as long as they consider it a righteous one. In like manner they even encourage the eating of meat by soldiers since it has been found that flesh makes them healthier, on the grounds that while it is bad to eat meat, it is worse that the right side should suffer on account of weakness arising from the lack of it. Thus, too, in the case of marriage, while Hinayana only allows it on sufferance since it knows that it is impossible to prevent it altogether, Mahayana, on the other hand, encourages it in the case of the layman in order to propagate a healthy race. In fact we even find some of the Buddhist casuists maintaining that the telling of a falsehood is justified in some cases, though, wisely enough, less emphasis has been laid upon this point than might be, owing to the fact that it would be popularly misunderstood and taken advantage of.

The Morality of the Salvation-by-Faith School.—It is necessary, before closing this chapter on Buddhist morality,

[21] For a striking illustration of the steady adherence of the Hinayana priests to the idea of non-resistance, see Fielding Hall, *The Soul of the People*, in the case of the British invasion of Burma.

to note something of the attitude of those sects of Mahayana Buddhism who profess the doctrine of the Pure Land. As we have already seen, they teach that Jodo or the highest heaven may be gained solely by faith in Amitabha (the Dharmakaya). Accordingly, to them asceticism in any guise is unknown, and like the Protestants, who, teaching practically the same doctrine as these sects, have done away with many of the fasts and penances of the Catholics, so has the Jodo or Sukhavati school (to give it the Sanskrit name) abnegated many of the practices of the salvation-by-works branch, and the most powerful branch of the sect in Japan, the Shin Shu, even allows its priests to marry and eat meat, etc.

It must not be supposed from this, however, that they have done away with morality in the ethical sense of that word, for we find them even more insistent than their brethren of the salvation-by-works school upon the necessity of every one, layman and cleric alike, leading a pure and holy life. With them, however, besides ordinary moral duties, there is the necessity of repeating the formula *Namu Amida Bu* or *Butsu*, frequently contracted to *Nembutsu*.[22] With one branch of this school the very reciting of the name is in the nature of a meritorious deed and accordingly whoever does so worthy of rebirth in the Pure Land, but in the Shin Shu the mere reciting is of no effect, but is simply a sign of faith in Amida, like the Protestant idea of the nature of good works after justification.

V. MAHAYANA BUDDHISM TO-DAY.

The Origin of Mahayana Buddhism.—Now just a word as to the origin of Mahayana. Many persons who are but

[22] The exact translation of this phrase is, *namu* "praise (to)," *Amida* a contraction of Amitabha, the Dharmakaya, and *Bu* or *Butsu* a contraction of Buddha, Amida being regarded as the great universal Buddha. Jodo priests and laymen go through their rosaries several times a day reciting this name, and the Chinese characters for the phrase are engraved at all sacred places of the sect, thus having the advantage over the Christian monogram, IHS, in that it can both be seen and heard.

little acquainted with the subject are under the impression that Mahayana as it is to-day, is a purely Chinese and Japanese product, or in other words, that it is the result of the failure of the Japanese and Chinese minds to understand or receive the doctrines of pure Buddhism as it was originally preached to them. This is entirely a mistake, for although Mahayana has acquired several new phases since its introduction into the Far East, yet we shall find that as far as all essentials go, the doctrines of Mahayana may be traced back to the native home of all Buddhism, India.

Buddhism was originally far more elastic than at present; accordingly we find in the first centuries after the death of the Buddha, widely different aspects of Buddhism existing side by side in a somewhat nebulous state. The bases of all later philosophies may be found there, but practically undeveloped, and often not clearly expressed. As time went on, these various phases of Buddhism gradually hardened and crystallized, the differences between the various schools were noted and harped upon, and we may notice the gradual drawing apart of several chief divisions. These divisions and schools are too numerous to be touched upon here, but they may be classified in a general way into the two main trends of thought which later developed into the schools of Hinayana and Mahayana as they exist to-day.

Early Patriarchs of Mahayana.—Since, then, Mahayana was a gradual development, and not the result of the teaching of one man or school (unless, of course, we admit the Mahayana claim and say that it was Gautama), it is obviously impossible to point to any one person as the sole founder. One or two names, however, may be mentioned as being those who were most influential in developing and spreading its teachings. Thus we have Parçva and Punyayaças, both of India. They are chiefly known as being the

teachers of Açvaghosa, from whose writings we have already quoted, and who may be regarded, more than any one else, as the father of modern Mahayana. Of Açvaghosa's life we have only conflicting and unsatisfactory accounts, but it may be said that he lived somewhere about the beginning of the Christian era and was originally one of the leaders of the anti-Buddhistic Brahmans; he was converted to Buddhism and spent the remainder of his life, honored both by the nobles and the people, in propagating the doctrines of Buddhism as he understood them.

As Açvaghosa presented them, however, the doctrines of Mahayanism were still somewhat indistinct and unformed; but two or three centuries (accounts vary, some say less than a century) afterward there arose Nagarjuna, also an Indian, who made it his life's work to systematize its doctrines, and under whose endeavors Mahayanism took practically its present shape. Mahayanists are fond of calling Nagarjuna the second founder of Buddhism. From this time on it is impossible to trace the history of the new faith in detail. About 53 A. D. Buddhism was first introduced into China, where in spite of various persecutions under antagonistic rulers it continued to thrive. While the doctrines of Hinayana found favor in Burma, Siam, and Ceylon, those of Mahayana, as they were gradually elaborated, appealed more to the Chinese tastes, so that in the Far East Mahayana continued to flourish long after the destruction of the mother church in India. Buddhism was introduced into Korea in the fourth century A. D. and into Japan in the sixth.

Division into Sects.—With that knack of elaboration and hair-splitting for which the Oriental mind is so noted, the doctrines which Açvaghosa could explain in a short monograph were gradually expanded until Mahayana came to include as it does to-day about 84,000 doctrines. Now it is manifestly impossible for any one organization to deal

equally with each and every one of such an enormous number of ideas. It must needs content itself with emphasizing one especial phase or aspect of the religion taken as a whole, while admitting in a general way the truth of the various other aspects. Accordingly we have the various sects and schools of Mahayana Buddhism, each one of which considers the phase which it presents as being of especial importance and the most vital and essential to enlightenment. We find in them elements appealing to the scientific, to the religious, the philosophical, the mystical, and the occult types of mind, to the educated and the uneducated, the deep and the superficial, and in the various sects which exist to-day, every person must find at least one which appeals to his own type of mind.

These sects are far too numerous even to mention them all, much less to go into details over their teachings, but a word must be said concerning the most interesting and important ones. Speaking generally, as we have already noticed, the Mahayana sects may be grouped into two main classes, those who follow the path of salvation by works, and those who adhere to the principle of salvation by faith. In China the latter, while influential, have not succeeded in gaining universal acceptance, but in Japan they hold the allegiance of about half the Buddhist laymen.

The Tien-Tai Sect.—Chief among the former or salvation-by-works school, comes the Tien-tai (Japanese Tendai) sect, which is one of the most scholarly and influential. It was founded in China, its head monastery being on Mt. Tien-tai (whence its name), but it early manifested a missionary spirit and after having quickly overrun Chōsen or Korea, it was brought over from there into Japan about 600 A. D., being the first Buddhist sect to be introduced there. Owing to the power of Shotoku Taishi it succeeded in gaining a hold in spite of early adverse circumstances. From that time on, both in China and Japan, it occupied a

very prominent place, many of the emperors and nobles of both countries giving it their allegiance, owing to which fact it may be said to occupy a position somewhat analogous to that which the Anglican Church would occupy if disestablished, though within the last few centuries it has somewhat declined in influence. Its principal Sutra is the Sutra of the Lotus of the Good Law (*Saddharma Pundarika Sutra*), but the sect is not noted for any special feature or doctrine, though maintaining in a general way the ideas outlined above. It may be regarded as strictly orthodox, and manifested a tendency to persecute the innovations which Shinran, the founder of the Shin Shu, strove to bring about.

The Dhyana Sect.—Another powerful sect, both in China and Japan, is the one that goes by the scholastic name of the Sect of the Buddha Heart, but which is popularly known as the Dhyana sect (Jhanna in Pali, Shan in Chinese, and Zen in Japanese). The word signifies meditation or religious contemplation, and from this something of the nature of the sect may be understood. It is often termed the Quakerism of the Orient, since it declares the Truth to be locked up in the heart of each man, and teaches that it can be unlocked by a proper system of meditation. While other sects found their teachings upon one or two Sutras, Zen professes to revere them all equally, or really to ignore them all equally, since it looks upon all externals, such as Sutras, etc., as being purely incidental (and often obstacles) toward the gaining of enlightenment.

One of the most noted things about this sect is its system of *kung-ang* (Japanese *ko-an*), or words or phrases with a hidden meaning which are given to each student to elucidate. One or two instances will give the reader some idea of their general nature. A monk once asked Tung-Shang (Jap. Donzan, 806-869), "Who is the Buddha?" to which the master replied, "Three pounds of flax." A some-

what similar story is recorded of Mu Ping (Jap. Mokyo) who when asked the first principle of Buddhism replied, "What a large melon is this." Each of these ko-an are supposed to be answered by one word. Here indeed is esotericism with a vengeance![23]

Bodhidharma, in the sixth century, was the first one to introduce this sect from India into China, where his fame soon attracted several influential followers, which insured its success. From China it was brought over to Japan in 1191. It is considered the most philosophical of the sects, and has all along held the allegiance of a large portion of the upper classes of both countries.

The Mantra Sect.—Of almost purely Japanese origin is the Mantra (or, to give it the Japanese name, Shingon) sect, one of the most mystical and hair-splitting of them all. It dates back to the time of Kobu Daishi, in the ninth century, who after mastering the principles of several other schools, felt that he had a revelation commanding him to combine the teachings of Buddha with those of Shinto, the ancient native faith of Japan. As the result of his influence, both direct and indirect (for his indirect influence was even greater than his direct influence), so thoroughly were the foreign and the native faiths intermingled that when after the restoration of the imperial power in Japan, an attempt was made to separate them, it was found almost impossible to do so, so much had one faith borrowed from the other.

The Shingon Shu bases many of its doctrines upon a supposed work of Nagarjuna (see above) entitled the *Mahayana-çastra-vyakya,* a deeply mystical work, but which outside of Shingon circles is generally supposed to be spurious, though it is certainly very ancient. As a matter of fact, Shingon approaches more nearly to Hinduism and Vedantism than does any other sect. Its most

[23] More about these *ko-an* may be found in the author's article "The Development of Japanese Buddhism," *The Open Court,* February, 1919, p. 107.

noticeable feature is its multiplicity of gods (most of them personifications) who are strangely and categorically arranged into thirteen main divisions, as well as the fourfold division of North, South, East, and West. In spite of this, however, Shingon is monotheistic (like the other Buddhist sects) in the sense that it admits that there is one supreme threefold Deity, an unmanifested God, sometimes called Abrakakia (corresponding to the Gnostic Abraxas), and a manifested God, Vairochana, corresponding to Amitabha (in Shingon, however, Amitabha is the title of a lower deity) or the Dharmakaya.

The Sukhavati or Jodo Sects.—Now we come to an examination of the salvation-by-faith-alone school. Researches have shown, contrary to general expectation, that the ideas presented by this school are considerably older than was at first considered possible, since we find them in their rudiments common in the times of Açvaghosa and Nagarjuna, and we often find these patriarchs preaching similar ideas themselves. The doctrine was then fully elaborated in China, where we find among others Zendo and Donran (to give them their Japanese names) bringing out the conception more explicitly.

It was not, however, until Buddhism was introduced into Japan that the doctrine as it exists to-day commanded any attention, when in the twelfth century we find Genku, or as he is sometimes called, Honen Shonin retiring from the Tendai sect to preach that for those for whom the narrow path of justification by one's own efforts was impossible, there was the path of justification by faith in Amitabha. Shodo-mon, the salvation-by-works school, taught that a man continued being reborn in this world until he attained Nirvana, which it must be remembered may be attained anywhere. Genku, however, while teaching that this was true, maintained that is was also true that

a believer might be reborn in the Pure Land, where enlightenment could be more easily obtained.

Genku retained the original discipline of Buddhism, but his illustrious disciple, Shinran Shonin, while changing but slightly the theology of his teacher, did away with the celibacy, the abstention, and the vegetarianism of the monks. Accordingly to-day, the priesthood of the sect which he started, the Shin Shu Jodo (True Sect of Pure Land) as it is called, is hereditary, the patriarchate of the various subsects passing from father to son.

The Nichiren Sect.—As was natural with the case of the Shin Shu, where the rules which he had laid down were abandoned, Gautama received but scant attention on their part, and his images were often cast aside. It was the sight of one of these battered images that was being played with by children in the street that proved to be the turning-point in the career of the famous Japanese priest, Nichiren, who was born A. D. 1222, the son of a fisherman. Except for this humble beginning, the life and character of Nichiren corresponds very closely to that of the Englishman Wesley. Nichiren after his conversion abandoned the more conservative and staid ways of spreading his gospel and revived the practice of vehement street preaching. Like his English parallel, Nichiren was somewhat bitter in his denunciation of the other existing sects, and alone among the present Mahayana schools, the sect which he started still maintains an attitude of exclusiveness and hostility.

Nichiren had little to add to the religious speculation of his time save a rabid zeal for devotional affairs. He assumed an attitude midway between the salvation-by-works and salvation-by-faith schools, declaring that works were of importance, but that the reciting of the phrase *Nan-myo-ho-ren-gekyo* was also of great benefit. With the Tendai sect, he took as his principal Sutra the *Sad-*

dharma Pundarika Sutra (the Sutra of the Lotus of the Good Law), but so much emphasis did he and his followers lay upon it that they are often called the bibliolaters of Buddhism. Nichiren has at present about two million followers, almost all of them in Japan, and they are chiefly known for their gaiety and light-heartedness in the observance of their religious customs in contradistinction to the somewhat graver behavior of their fellow Buddhists.

The Relation of the Sects.—Such, then, are the more important and interesting of the sects of Mahayanism. In addition to these, there are also numerous others, but the latter are either of too little numerical importance or else their doctrines are possessed of too little distinction to warrant special attention. Furthermore, there are in Japan and China three Hinayana sects. These have neither priests nor temples but are merely studied by the other sects. Finally, there is Lamaism, the religion of Thibet, differing in many respects from the other Mahayana sects but lying outside the scope of this discussion. In Japan all the sects are still flourishing, though they have practically all broken into smaller subdivisions, but in China, where Buddhism is at a much lower ebb than in Japan, the only divisions that exist are between the priests of the Yellow Robe, in which are combined more or less perfectly the different sects which have been enumerated above, and those of the Blue Robe, or the Lamaistic priests, who acknowledge the Grand Lama of Thibet as their spiritual leader.

As has already been seen, however, while Mahayana is divided into many sects and factions, yet almost universally do they give recognizance to one another, each claiming for itself only that it emphasizes the more important features of the Mahayana faith. Even the schools of salvation by faith and by works differ but little in reality, since the Sukhavati sects teach that while by faith a man may gain admittance into the Pure Land after death where it is

easier for him to realize the "truth which shall make him free" (to use Christian verbiage), yet must he come back to the world for the salvation of all; the opposite party teaches that while a man is constantly being reborn on earth, yet must he finally reach the Pure Land and Nirvana. They are but two ways to the same end, says the Buddhist.

The Present Position of Mahayana.—A word must be said before closing concerning the present condition of Mahayana Buddhism. Soon after the opening of both Japan and China to the world, Buddhism underwent a gradual but rapid decline both on account of the Christian missionaries on the one hand and of materialism on the other. Many of the monasteries were deserted and were sold or turned over to the government to be used for other purposes, and it seemed to all parties as if the days of Buddhism were numbered. Within the last few years, however, a reaction has set in, a reaction which promises to be quite as radical if not more so than when the pendulum swung the other way. When Christianity first entered China and Japan, Buddhism had, to no small extent, sunk into decay. Lately, however, with the advent of opposition, has come a revival of the old missionary spirit, and accordingly we find Buddhist schools and colleges being erected, newspapers and magazines started, and extensive missionary courses given, which have resulted in an increase of religious fervor and piety. In addition to all this, we even find at times extensive plans being made for missionizing abroad. It is impossible, of course, to tell what the future of this new movement will be, and possibly the revival of the Buddhist faith may be illusive and temporary, but certainly if present indications are correct, the Phœnix of the new Mahayana Buddhism is arising from the ashes of the old.

Wm. Montgomery McGovern.

Kyoto, Japan.

THE SECRET.

THAT which was in the beginning is neither young nor old;
It started not; it was formless thing, with no texture or fold.
'Twas endless unlimited mass, without distinction of parts.
It never engendered the maddened stroke of lightning which darts
With power formidable into places that knew it not;
Unmoved it was, nor good nor evil, nor cold nor hot.

No longer this substance stands in its pristine state apart;
It yields to the hand of man and is moulded by his art
To shapes which please his fancy or serve the purpose he forms.
'Tis stirred by the ebb and flow of movement, and heavy storms
Now beat upon it part upon part.—What we see grows old;
That which was in the beginning is now lost from our hold.

Through ages long and by changes many and moving slow
Was part from part separated, becoming stone or snow
Or air or man; and whence or by what process they came
Is not revealed in the forms that now our notice claim.
No memory reaches back, and searching alone can find
The intricate secret which is hid from the view of mind.

In Man yet deeper grows the longing this secret to know.
To-day he labors with power, and not is his progress slow:
The past he penetrates further; forth he reaches his hand;
The truth he will find if record there is on sea or land
Or in the firmament, or the future can make it known
By throwing its light where once the light of the past hath shone.

—O, hasten, glad Day, when Man by search this knowledge shall find
And matter shall yield its longest-treasured secret to mind.

R. D. CARMICHAEL.

SCIENCE AND RELIGION.[1]

IF science and religion are to continue to coexist it seems opposed to the conditions of modern thought to admit that this result can be brought about by the so-called "water-tight compartment" system which, even at the present time, is frequently extolled or considered possible.

On the one hand, indeed, modern science, after invading the domains of psychology, sociology, and morals, lays claim to study and examine every form of human life, without exception.

On the other hand, religion is conceived either as pure feeling or as a blend of feeling and knowledge. In the former conception, science regards it as but a crude datum which it must explain in accordance with its own principles, by bringing it analytically within the compass of scientifically known phenomena. Again, if it participates in knowledge, it cannot be radically isolated from science, for we cannot conceive of the possibility of two wholly heterogeneous truths.

In the following, science and religion are to be brought face to face with each other. The question, however, of the conditions that would bring about an intelligible relationship between two really distinct terms, is anything but simple. Identity is only a logical fiction; non-contradiction, a limitative condition. On the other hand, simple juxtaposition forms just that empirical relationship which

[1] [Argument for discussion by the *Société française de philosophie* at a meeting held November 19, 1908. Authorized translation by Fred Rothwell.]

we have here to supersede and, if possible, exalt into one that is intelligible.

Philosophy has been seeking, ever since the time of Plato, in Descartes, Leibniz, Kant, and Hegel, for a kind of connection which should be alike concrete and intelligible. Besides, there is nothing to prove that there exists only a sort of concrete relation: it is quite possible that there may be between religion and science a connection that is both intelligible and *sui generis*.

II. What, exactly, are the terms we have to bring together? In these days it is evidently less science and religion as objectively presented systems, than the scientific and the religious *spirits*. It is the scientific spirit which, by its own methods and from its own point of view, leaving out of account the results at which it arrives, sometimes regards itself as the negation *a priori* of the religious spirit. The latter in turn would regard itself as abdicating its position did it admit that its very existence could wholly be explained by scientific methods.

But do not the scientific and the religious spirits, when confronted with each other, appear to be in a relation of strict contradiction—a fact which, in the eyes of any man who respected the fundamental laws of logic, would immediately settle the question? Such a relation of contradiction might have existed between a scientific spirit imbued with dogmatic metaphysics, and a religious spirit identified with the letter of any positive religion. It is but right, however, to distinguish between the dogmatic scientific spirit and the strictly scientific spirit, or the sense of the superiority of experience; and also between the materialistic religious spirit and the strictly religious spirit, or the sense of the supremacy of the ideal.

Between the scientific spirit and the religious spirit, both strictly so called, there is no logical contradiction Still, might not there be heterogeneity, pure and simple?

Shall we find the *intelligible relationship* between these two terms which we regard as demanded by the modern mind?

III. In what is strictly called reason, we possess a principle of judgment which enables us to give an impartial estimate of religion and science in so far as reason, in the concrete elements that distinguish it from simple logical mechanism, is made up of those data of science and life which the human intellect has comparatively succeeded in universalizing. And so it is for reason to examine the problem, and from its standpoint we now consider the matter.

1. Is science self-sufficient?

It is, practically speaking, from the scientific point of view. From that of reason, strictly so called, it is not, because it has for a condition that things should be given, and given things presuppose activities that produce them. Besides, science is not mechanically produced in the human mind; it presupposes original investigation, and this also is activity.

Science presupposes life.

2. What are the conditions of life, or more precisely, of human life?

For man, a being endowed with conscience, this problem not only signifies: In what conditions can human life, such as it is, exist?—it also signifies: In what conditions is a strictly human life possible and can it acquire all the power and excellence of which it is capable?

Reason may ask the second question as well as the first.

If, after asking this second question, we try to find its solution in history and in the human conscience, we see that a strictly human life, the life that tends to all that is best, necessarily implies, (1) a faith; (2) the representation of an ideal; (3) a sense of love corresponding to this faith and ideal. These three elements, when precisely ana-

lyzed, appear to be extra-scientific. Now it is just these which with ever-increasing distinctness constitute the form of the religious spirit.

Religion, then, is the higher motive principle of the human soul, the principle which enables it to transcend itself.

3. Religion is connected with science through the medium of life. Science brings reason to admit the reality of life. Religion supplies man with the means of imparting to his life the utmost possible power and value. In man, as he is presented to us, the will to seek the best has its principle, immediate or mediate, perceived or unperceived, in determinations which are the province of the religious spirit.

Thus the relation of religion to science is neither analytical nor synthetical. It is a relation that is contingent and yet rational. It is a progress wherein liberty is entreated without being compelled. Strictly speaking, it is a passing from fact to action or true being, and from being to power and to what ought to be.

IV. Is this theory satisfactory, respecting the scientific and the religious spirits?

1. In the name of science, the objection may be raised that while an incomplete science admits of the conception of activity, of the possible and the obligatory, as veritable realities, a complete science would reduce these notions to subjective illusions.

But can there be a complete science? Is being from all eternity a thing that is made, with the result that, once all the laws of nature are known, nature has nothing more to do than conform to the dictates of science? Or does being really come about with time, so that it is forever indispensable to observe it in order to become acquainted with it? It seems as though we should be begging the

question if we expected being to conform to science, and not science to being.

2. In the name of religion, the objection may be urged that the given religions contain many other elements than those just mentioned, and that to establish the legitimacy of religion in its purely formal aspect is but a poor justification of it.

It is clear that faith, the representation of an ideal, and love,—which are simple forms of will, intelligence, and feeling,—are by themselves only a frame in which each nation and individual, according to knowledge and character, tends to set a content increasingly true and worthy as well as capable of determining human life.

It remains certain, however, that the matter of given religions presents a truly religious character only if it is in harmony with this form. To have religious ideas, it is not sufficient to utter the name of God; while, on the other hand, it is possible to retain the thing though its name be changed.

The pure form of religion is still but an abstraction, because it is only those things that can be grasped by sense or imagination which appear to us as real. The letter, however, though indispensable to the spirit, is not to be confused therewith; it is a variable compromise between the spirit and the changing conditions of the environment in which the spirit must find its realization.

I.

Is the question of the relations between science and religion, which has exercised the human mind during the past few centuries, still a living one? Does it not now seem to consist of a rivalry between persons and powers, rather than of a genuine conflict of principles? I imagine that many, nowadays, are inclined to believe that we have

at last found the magic syllables which must insure peace once for all, in the word *separation*:

"Wenn sich zweie lieben sollen,
Braucht man sie nur zu scheiden."

Science, it is said, comes within the sphere of reason, and religion within that of feeling. Why should there be any conflict between these two powers—alike essential—of the human soul? If religion remains pure feeling, and science restricts itself to storing the mind with the laws that control the mutual relations of phenomena, then we cannot see how religion and science are ever to meet. They occupy two worlds outside of and alien to each other.

Up to a certain point, this separation may be realized in a society. It is seen to exist, indeed, in the consciousness of certain individuals who, in accordance with physiological, if not logical, laws, simultaneously accept different principles which they do not compare with each other. Still, in theory and for the future, the so-called "water-tight" system is no longer adequate to decide upon the relations between science and religion.

We live in an age of world-wide communication and comparison: nations, social classes, literature and science, art and life, society and individual, certain sciences set over against certain others, respectively clash and struggle for existence. Religion cannot possibly prove an exception to this law and stand unshaken by simply ignoring in the world the existence of everything save itself.

In more exact terms, science having successfully applied itself to biology, psychology, and sociology, as well as to mathematics and physics, perceives no limits to its sphere of influence, and professes to explain, along the lines of its own determinism, the appearance of religious as well as of all other phenomena. What matters it if religion exists wholly in feeling? A feeling is a mental

state, consequently an object whose conditions of existence come within the scope of science. Science will demonstrate why, given my temperament and my environment, I was bound to have certain religious tendencies, just as it demonstrates why I was bound to incur some particular illness.

Religion, on the other hand, driven back into the inmost depths of consciousness, becomes so feeble that it can no longer be recognized. What is there in common between this flickering, self-centered spark, afraid of being seen because an effort would at once be made to extinguish it, and the all-devouring fire which once professed to inflame the universe: Γενηθήτω τὸ θέλημά σου ὡς ἐν οὐρανῷ καὶ ἐπὶ γῆς!

Every true religion, in the first place, possesses an intellectual content, and in the second place, aims at manifesting itself in the world.

It may be that, in certain individuals, a radical dualism of religious faith and scientific knowledge is maintained, just as in certain treatises on psychology we find the material separation of the faculties; but neither the human consciousness, taken in its essence, which is unity and harmony, nor our modern societies, which by means of journals, reviews, and meetings of every kind are constantly bringing different men and different doctrines in contact with each other, will indefinitely maintain the coexistence of science and religion unless there exist between the two, intelligible relations of compatibility and harmony.

* * *

What method must we follow to discover whether it is possible for science and religion to harmonize in an enlightened and introspective consciousness?

It is possible, following along experimental lines nowadays so prevalent both in philosophy and in other sciences, to set forth a method which consists in taking a few re-

markable instances of individuals who have reconciled and are now reconciling strong religious convictions with the cult of science, and trying to find by a process of analysis how in the consciousness of these men the reconciliation between the two disciplines comes about.

This purely descriptive method, however, will not supply us with results apt to influence our own conduct unless the reconciliation effected in these consciousnesses appears to have real value, not only from the point of view of the individuals in question but in itself, in other words, an objective value.

In order to find out objectively what is the relation between science and religion, shall we appeal to history? Will the study of the origin and of the vicissitudes of religion and science determine what it is that constitutes the essence of both alike?

But, then, according to the law of evolution which nowadays is held in such high esteem, both may have changed very considerably, and if we are dealing with ourselves here and not with others, if we inquire at the present time with what conclusion we are to side in order to live a life that conforms with the present state of the human mind, we shall require to consider religion and science as they appear at the present day, not as they may have existed among prehistoric peoples, or even in a society that immediately preceded our own.

What does this mean except that we must deal with the problem, not from the point of view of descriptive psychology or of history, but from that of reason?

We have only to arrive at an understanding as to the meaning of this word: reason. Is it the sole characteristic of reason to reduce the relations of difference given to us in nature to relations of identity and contradiction, in such a way that, if two things cannot be reduced to one and the same essence, they may be declared incompatible? If this

is reason, it will certainly be difficult to maintain the equal right of science and religion to existence. On the one hand, science and religion cannot be identified without being destroyed: however we interpret them, science is the explanation of things by things themselves, religion is faith in a relation of things to some higher power.

If then, in the sight of reason, radical irreducibility means a relation of opposition by contradiction, then science and religion are incompatible; reason insists on the one disappearing in presence of the other: "This will kill that!"

Science, moreover, exists and continues henceforth in all probability like nature itself, of which it is but an abridged and practically utilizable representation: *it is*; whereas religion cannot be conceived apart from faith in some thing which neither is nor can be given in nature.

If, then, reason is but the faculty of reasoning according to the principles of identity and contradiction, it seems difficult not to recognize that from the standpoint of reason religion is doomed to disappear before science, sooner or later.

But in reality reason shows us other relations than those of logical identity and contradiction. In the name of reason, what philosophers have always attempted to define is just that faculty, peculiar to man, of attributing a reality and value to relations which cannot be reduced to logical identity and incompatibility, purely and simply. Metaphysics is strictly the superposition of relations of this kind on simply logical relations. Such are the μέθεξις or the κοινωνία of Plato, the finality of Aristotle, the conjunction of terms in the intellectual intuition of Descartes, the moral necessity of Leibniz, the synthetic judgment of Kant, the synthesis of Hegel. Unlike science, whose ideal is to reduce the diverse to the identical, metaphysics attempts to find intelligible relations, relations of conformity between

different beings, even in so far as they are different and logically irreducible.

Besides, there is more than one sort of relation calculated to satisfy that faculty—the living sum total of the experiences and reflections of mankind—which we call reason. It is not by presenting *a priori* a definition of rational relationship, and then trying to find out syllogistically whether this definition applies to the given instance, that we can discover the rational relations between existing things: it is rather by engaging in an attempt to bring the real and the rational face to face and mutually determining each other. To understand is to adapt the reason to the object quite as much as to bring the object within the laws of reason.

II.

As regards science and religion, what exactly are the terms we have to bring together?

These cannot be science and religion regarded as actual bodies of doctrines.

Science at the present time is no longer carried on deductively but rather inductively. It follows that none of its principles, not even the most general, are acquired in any final way. To exalt into a dogma any one of its present conclusions and set it over against religion, would be unjust. And it would be quite as unjust to argue that because science is always capable of improvement, we need take no notice of the difficulties it raises against religious affirmations. It is useless to say: religion is invincible, since it upholds its certainties where science acknowledges that it in turn affords only probabilities. The acquisitions of science are final, in the sense that its future progress will simplify, put in its place, absorb and supersede, but not destroy the experimental content of its present assertions. None the less is it a fact that a complete and final science

is an idle dream. Science is nothing else than the continual progress of the human mind, seeking for the explanation of things in things themselves.

And no less chimerical would it be to compare religion, as an *ensemble* of positive dogmas and determinate practices, with science as it is at present offered to us. In this case, which religion should we take? What value would the harmony or the disharmony established between science and some particular religion possess in the minds of those who profess a different religion? For even in these days, when we consider both dogma and practice, there is a striking difference, in the very heart of Christianity itself, between a Scotch Presbyterian and an Orthodox Greek.

Again, even if we thought we had set up an agreement between dogma and science, should we not in this be deceiving ourselves? Have words the same meaning in the religious as in the scientific order of things? If the man of science admits that there is no scientific law incapable of being belied by facts, does he thereby mean that science recognizes the possibility of a miracle as the production of a fact outside of all physical laws? Or does he not simply mean that, along with the known laws of our science, there may be cooperating others which are still unknown, and which may thus determine results that are at present inexplicable?

Neither the harmony nor the disharmony of present-day science with any particular body of religious doctrines, when we come to analyze it, has any real philosophic bearing.

At bottom, the conflict is far less between science and religion as given facts, than between the religious and the scientific spirits.

The scientific spirit consists in eliminating from the explanation of a given phenomenon, every cause that is not itself phenomenal, in explaining facts by facts, nature

by nature. Science has made the discovery that, in nature, not only are there laws or relatively permanent connections between facts, but that these laws come under one another so that we may set up the ideal of bringing them all under a single one.

Religion requires that each phenomenon, taken separately, shall be the object of a special decree emanating from a supernatural power, and that nature, in some of these phenomena in particular, shall testify to the operation of forces which do not belong to nature. For science, being is a unity and it is nature that is this being; for religion, being is a duality: it includes, apart from nature, God, who transcends it infinitely.

* * *

Such is the real conflict, and we must recognize that it is far more profound than the opposition between a given scientific doctrine and some particular religious dogma would be.

It is not difficult to set forth the relation between the scientific spirit and the religious spirit as forming a regular contradiction. According to this view, we may say that science is the negation of the supernatural, and religion its affirmation; science depersonalizes all things, religion looks upon personality as their foundation. Science is explanation, religion is mystery.

It is clear that if, yielding to a taste for logical contrasts, I define nature as a system of mechanical forces and life as a spontaneity, the statement that there should be life in nature appears to be contradictory. For all that, life does exist in nature. We must beware of creating, by means of our concepts and verbal antitheses, contradictions which possibly correspond to nothing real.

What, then, are the scientific spirit and the religious spirit in reality, not according to some academic concept or

other? Up to quite recently, the scientific spirit might have been defined as the conviction that things are reducible to universal mechanism and determinism. Nor is it even nowadays a rare thing to hear religion defined as a belief in the possibility of miracles as an infringement of the laws of nature.

If we define religion and science in this way, no doubt the difficulty is irremediable. Indeed, the one denies the very thing which the other affirms. To blend them in one and the same consciousness, we must abandon the principle of contradiction.

But then, do such definitions still correspond to reality? Science is at present essentially experimental; hence it appears as though the mechanical form of determinism which was considered inherent in the laws of nature itself, were but an abstract conception, added by the dogmatic metaphysical spirit to the real laws of things from without. A science that affirms mechanical determinism as being in things themselves and not merely in a certain metaphysical conception of things, ceases to be simply experimental: it is a philosophy of science, not plain science alone.

On the other hand, the simple statement of the extreme variety of religions compels the philosopher to distinguish between religions and religion, and to endeavor to find, by the aid of reason, what there is truly characteristic and original in the religious spirit.

The idea that religion is essentially the belief in a power which at one time leaves nature to follow its course and at another opposes it, is inseparable from a conception of science according to which we should be enabled to know certain natural laws as absolutely simple, primary, and universal. This way of interpreting natural laws is nothing else than regarding the laws of nature as belonging to the same category as laws decreed by a legislator. In this latter case, the law is first decreed, and afterward either

obeyed or violated. Scientific laws, however, are not prior to facts; they recapitulate them. If a law is violated, it is not because an opposing power has intervened, it is either because the law in question did not possess the generality attributed to it, or because some other law happened to be involved in the phenomenon, or again because the latter does not come under our present scientific categories. Laws are devised for the explanation of facts; facts are not like those examples in old grammar-books, showing us the language as a rule obedient enough, though at times quite opposed to the laws and precepts of the grammarians.

Along with the transformation of dogmatic into experimental science, there disappears the notion that a miracle is an outrage upon nature; religion can no longer consist in devising and coaxing forces which are stronger than those of nature, which are therefore homogeneous with these, therefore natural still, and which have, in reality, the same claim to come within the compass of science as have laws actually discovered. The force that produces thunder was supernatural so long as it could not be brought within the compass of electricity.

As against modern experimental science, the supernatural can no longer be interpreted in a material sense, thus regarding it as a still natural and simply unexplored essence. It can only really be distinguished from nature if it is strictly spiritual, that is to say, if it is the expression of spirit as life, action, creation and realization of the true, the just, the beautiful, the good and the ideal, in opposition to matter as the supposedly permanent, homogeneous, and identical foundation for the sum total of phenomena.

In other terms, whereas the principle of science, whose object is to determine, classify, and systematize the relations of things to one another, is the supremacy of experience; the principle of religion is the supremacy of the ideal. The religious spirit believes in the existence, the

autonomy, and the power of the ideal, along with all those attributes that we regard as necessary for its existence and efficacy.

Thus conceived, the religious spirit and the scientific spirit evidently do not hold to each other that relation of logical incompatibility apparent when they are both interpreted in a dogmatic and materialistic sense.

Science takes into consideration objectively given phenomena and their mutual relations: it knows nothing of their generating causes. It is based on the postulate that it is useless for us to inquire into the cause that produces phenomena, in order to be enabled to connect them with one another, to classify and systematize them; along these lines it systematizes only so far as observation will allow.

Religion is connected with the springs of being. It is not simply a moving body of feelings, an instigator of actions; it is itself feeling, thought, life, action, the direct relation of souls to one another, the converse and communion of spirit with spirit.

Hence it does not interfere with science, for, in whatever way its action is expressed in the world of phenomena, it admits that its external manifestations shall be objects of science, exactly like all other phenomena. It does not inquire what will become of the scientific explanation of the final residua of its actions. How can the brain that generates thoughts be concerned regarding the name of the chemical substances which will be found in its ashes?

There is no contradiction between the religious spirit and the scientific spirit, as men have imagined. All the same, we have stated that, in order to maintain equally two different principles, it is not sufficient that human reason should see them actually coexist, it must discern between them some intelligible and rational relation which will make of them not simply two objects set alongside of each other, but the conjoint elements of one and the same

harmony. Is there such a relation between the religious and the scientific spirits, that is, a relation not only of non-contradiction, but of mutual conformity?

III.

What, exactly, is the point of view from which we must examine this question? It can neither be that of science nor that of religion; for then the problem would already have been solved, though in two contradictory ways. Is there a third point of view?

That of reason, strictly so called, appears to satisfy the conditions required. Reason is not the principle of religion, for it is essentially intellectual and universal, whereas religion is life and personal consciousness. Reason cannot be confused with the general form of science, for the latter tends to reduce the diverse to the identical, whereas reason aims at maintaining and reconciling, while respecting their diversity, all modes of being which seem to it to possess real value.

What is reason?

It is but too evident that it is not simple logic. Logic is relative: such or such a principle being admitted, by hypothesis, it is logical or illogical to deduce therefrom such or such a conclusion. Reason comes to an immediate pronouncement: this may be logical but it is not reasonable.

Reason is not an *ensemble* of innate, immutable principles, laid down *a priori*, as dogmatic metaphysicians supposed. It is not the revelation, inscribed on the tablets of our consciousness, of a transcendent stereotyped truth that has come down to us from all eternity. It is not ready-made but makes itself, and has a history of its own. It is formed, as Descartes saw, of the scientific knowledge and practical experiences on which our intellect is nourished. It is that portion of the data of science and life

which human reflection, speaking relatively, has universalized.

* * *

If reason is such, is there any connection between science and religion, from this point of view?

Take science, first, for it is science that at the present time demands the assent of our reason immediately. In scientific experience, strictly so called, it possesses a material guaranty of universal value which cannot be found elsewhere.

Science, from its own point of view, is self-sufficient. For its self-construction it has nowadays nothing to do with transcendent principles or ends. It is reduced to a mode of determining facts and their observable connections. Does it follow that it is self-sufficient from the point of view of reason?

Science accepts things as given; its duty and honor consists in absolute submission to the conditions of this given. Being fundamentally experimental, it will always refrain from pronouncing upon the first elements of this given, or upon the principle according to which this given changes. It is indebted for its very existence to the strange circumstance that certain aggregates, certain complex connections, are in nature relatively permanent. It has to do with—and can only have to do with—states, not with true causes.

It therefore follows that, in the sight of reason, science necessarily posits the existence of being as the model, radically distinct from itself, of which it endeavors to give us a translation that can be utilized, from the human standpoint. An adequate science, one convertible with being, appears a contradictory concept, seeing that science has become essentially experimental.

Nor is this all. Precisely because our science is other than being it requires expedients, modes of representation

which, in all their elements, are not supplied by being itself. We are no longer living at a time when science sprang fully armed from facts, like Minerva from the brain of Jupiter. Science is concerned only with facts, but it is the human mind that constructs it. The human mind builds it up from concepts, signs, and symbols which it invents in order to deal, in its own way, with objects heterogeneous to itself. It imprints its stamp on facts.

And so, just as science presupposes being and is unable to reduce it, it also presupposes spirit of which it is the product. It is the living relation, daily more strict and subtle, of spirit with being.

What does this mean except that, from the standpoint of reason, science presupposes: externally, a creative activity that supplies it with matter for observation; internally, a similarly creative activity which contrives workable symbols, fitted to represent inaccessible realities.

Science presupposes life, both without and within ourselves.

In its turn, what is presupposed by the life of which we are conscious, that which concerns us directly, human life?

There are things without which we *cannot* live: in so far as we seek after these things, our life does not transcend in dignity the universal animal life. There are things without which we *do not will* to live: strictly speaking, it is the search after these things, materially useless, that stamps us as human beings.

What are these special conditions of human life?

Man seeks after certain things because he attributes to them an intrinsic value or worth. He does not consider them good simply because they either are or appear to him to be good. He subordinates himself to them, making his own goodness consist in adapting himself, submitting and sacrificing himself to them. The most precise manifesta-

tion of this faculty is the sense of duty, which is found to be the origin of every heroic act of the human will, and the disappearance of which cannot be imagined without our seeing human life change its aspect and fall back into a state resembling pure animality, from the moral point of view.

Now, the notion of absolute value, the notion of duty, cannot be reduced to strict knowledge: it is a matter of faith. It implies a risk, an adventure, a sort of logical absurdity. I do not know if what I attempt is possible, nor if myself and my fellow-beings will profit by it, but I know I ought to act so, whatever the consequences. To characterize such certainty, there is no other word than that of faith.

Still, while I believe in the duty of pursuing certain ends, I inquire of what it is exactly that these higher ends consist. Faith necessarily tries to attain to the knowledge and understanding of its object.

We may give the name of ideal to that supreme object, the pursuit of which is the essence of human activity. Each of the determinate forms we set up as an object to pursue is an intermediate term between reality as we see it and the supreme ideal. This latter is conceived by us as the perfection of truth and beauty, justice and good, consciousness and personality, in so far as this perfection is not only an abstract possibility but a necessary reality. The ideal is that which connects perfection and existence, their indissoluble unity.

Man, in a word, cannot limit himself to a contemplation of the ideal by his intellect, and a submission to it by his will. His union with the ideal can only be consummated by love, and the highest of all duties is that of loving. The pursuit of the ideal, individual perfection itself, presupposes the common efforts of human individuals. Not one of them could, by his own strength alone, become all he is

capable of being, and do all he is capable of doing. The common love of the ideal and, at the same time, the love of men for one another, is a third trait of a fully human life.

Man, if he is to be fully man, must not be content with living, he must devote his life to the practice of duty, the search after the ideal, the communion of one consciousness with others in love.

Are these three objects: duty, ideal, and love, strictly natural?

Assuredly they are conformable to reason, which, of itself, perceives their necessity for the human soul. But then, our reason is a state, an actual form of our intellect, it is not a first principle. To say that a belief has its original cause in reason, is to act like a psychologist attempting to discover the origin of the various languages in the very rules that grammarians deduce from them.

It is remarkable that those objects which the reason regards as indispensable to a truly human culture, are just those declared to be preeminently religious objects in that one religion which is considered as among the greatest.

What is the greatest commandment of the law, Jesus is asked. He replies: Thou shalt love the Lord thy God with all thy heart, with all thy mind, with all thy soul, and with all thy strength: this is the first and greatest commandment. And the second is like unto it: Thou shalt love thy neighbor as thyself.

Duty, belief in God, love: the commandments regarded by Jesus as preeminent contain these three elements, and these three alone.

Now, the work of Jesus is manifestly a religious work.

And, indeed, if these three objects form only a part of what the religious consciousness generally comprises in the idea of religion, none the less do they come in reality within the scope of religion, strictly so called, and not of simple morals, or what is called natural religion.

The sense of duty may be recognized by the psychologist or the moralist as actually present in the human soul; but where does it come from and what is it worth? Kant himself sees that these questions cannot be answered by the human intellect, whose function it is to explain fact by fact, being by being: here we have to find the springs of being.

Similarly, the concept of God is that of an unobservable reality, affirmed to exist, from the very fact that it is outside of all knowable conditions of existence.

And love, deep within the life of the individual, is the creation of a fuller life, in which divers individuals hold communion and the personality, far from being effaced, becomes more distinct and original.

It is indeed possible, by changing the meaning of the words: reason and nature, to maintain that these phenomena are also explained by reason alone and nature alone, just as it is possible, in another way—concluding, like Saint Augustine, that man possesses nothing but what he has received—to attribute to divine action all that man imagines he thinks and does of himself.

In fact, we find that the historians of philosophy are for the most part more inclined to explain such concepts as the categorical imperative of Kant or the intellectual love of Spinoza by an influx of religious tradition, than to account for the evangelical doctrine of the gift of oneself by the reflection of natural reason alone.

This is because the notions of duty, ideal, and love bring into human life a profound dualism which is alien to the simple idea of nature and reason.

To try to find, apart from being, the way to modify being and make it more perfect, is the property of religious thought. If we insist on giving the name of rational thought to this thought, then, by an abuse of language, we are calling reason the very thing that religion calls faith, grace, God, Providence.

Thus, through the medium of the notion of life, religion is connected with science. The man who reflects on science is led to attribute a reality to his own life; and the man who would fully live the human life must believe in duty, hold up perfection as his model, and devote himself in love. Now, acting in this way, he opens up a way in his soul to what is called religion.

Thus interpreted, can the relation of religion to science be called rational?

Evidently this relation is not simply a logical one. Science neither contains religion nor does religion contain science. The transition from science to religion is possible; it is not necessary.

Is it then but a simple conjunction of fact, based on the ascertained statement that certain men, in the course of their life, associate evangelical dreams with the pure and scrupulous cult of science? The deduction we have just attempted goes beyond such a result as this. Whereas science considers that which is in its actually given being, religion goes back to the origins and initial determination of being. Its objective is the ideal and the possible, what ought to be and the realization of what ought to be.

Now, the passing from what is to what ought to be is not logically necessary, neither is it logically arbitrary and irrational; it presents that solidarity in contingence which is actually the kind of connection that reason applies itself to determine. It is a special relation, perhaps a unique one of its kind. Now, it is too simple a matter to deny the existence of a relation under the pretext that we see no way of reconciling it with the logical relations, strictly so called. Life is made up of relations that are extra-logical, and yet real. It is such a relationship that unites science and religion. Though particularly difficult to define, it is better to attempt an imperfect definition of it than to deny it *a priori*.

The relation of science to religion recalls, we might say, that of actual language, relatively fixed and determinate, to living thought, which is its soul and origin; or rather the relation of an artist's work, apparently endowed with an existence and a unity of its own, to the creative and free genius which gives this work not only its existence, but the very unity and logic that characterize it.

IV.

It may be that objections will be raised to the theory here set forth.

Scientists may allege that, though an incomplete science always leaves outside of itself some empty space into which religion may find its way, science *per se*, as a whole, explains everything, fills all the regions of being, and in consequence, relegates religion to the realm of idle fancy. In theory, we may say, if not actually, science resolves into data of experience the concepts of the possible, obligation, value, the sacred, the infinite, religious faith, feelings of divine love, divine presence and activity, as well as the chemical, physical, and mechanical forces. Science regards the word "inexplicable" as having no other meaning than "not yet explained."

Such, in effect, is the postulate of science. It is from this point of view that it interrogates nature. Still, a question is not an answer, and that the solution should do away with the datum is unintelligible. An experimental science presupposes a being set up for its observation; all it can affirm is that, at some particular moment, it has grasped all the generating principles of the things under investigation, and no longer needs to observe them.

Besides, how could it affirm that the future of being is wholly predetermined, from this time forward? Science, such as the progress of philosophy has made it, dwells on

uniformities, analogies, correspondencies, but it neither attains nor seeks to attain to the origins and conditions, either of uniformity or of change, that reign throughout nature. It neither knows nor wishes to know if being is limited to being, or if it lives.

Science *presupposes* being, it cannot be substituted for it.

* * *

No doubt, also, we may foresee an objection on the side of religion.

Is it indeed religion, effective and concrete, with its own distinctive and specific characteristics, whose legitimacy has been established in the foregoing deduction? Have we really superseded so-called natural religion, so inadequate to the religious mind, or even simple rational morals?

It is somewhat difficult to reduce to unity or to a few leading points—so as to grasp and examine them all together—the objections that may be raised from this point of view. What are the component elements of a positive religion? The answer will vary according to the religion professed, each person being inclined to look upon all the parts of his religion as equally essential. Where is the line of demarcation between a natural religion and a revealed religion? What is the supernatural, what is revelation? Is the religion which, according to the preaching of Christ, may be summed up as love of God and love of one's neighbor, a natural or a revealed religion? Is the origin of the myths and rites, with which the religions of old were overburdened, to be found in genuine supernatural revelations or in the workings of man's imagination?

There is a mode of insisting that a religion shall contain revealed and supernatural elements, which tends to nothing less than the downfall of religions, the supernat-

ural postulated being actually one that is decidedly incompatible with the conditions of modern science.

And there is a system of morals, a religion presented as natural which in reality is more than a mere system of morals, more than a simply natural religion, because it contains principles that cannot be reduced to being as given to us by experience, because it implies a supernaturalistic conception of nature itself. When Pascal writes: "Man infinitely transcends man," he means that in human nature itself, examined in its inmost depths, we find the supernatural and the stamp of divine activity.

Faith, the ideal, love, in their true meaning, are indeed supernatural elements, in the meaning that a sane philosophy attributes to this metaphor. And it is into the sphere of religion, not of morals only, that we are brought by the precept of Jesus: Thou shalt love God above all things, and thy neighbor as thyself. To give oneself, to sacrifice oneself, if necessary, in order to further the coming of an ideal kingdom, to draw one's strength, joy, and being from an invisible being whose existence our senses deny, how could such conduct be anything but religious? What other activity, distinct from this, is more religious?

Besides, the theory here set forth does not aim at reducing religion to its purely formal elements, and excluding its positive determinations, its dogmas and rites; on the contrary, it finds its natural complement in the other theory, that form demands a formula; it abides by the profound saying of Fichte: *"Die Formel ist die grösste Wohlthat für den Menschen."*

This should not be taken to mean that the concrete formula, matter, is simply to be deduced from form; a thing that can scarcely be conceived, and which Kant himself really never thought of doing.

Though the form of religious life is in some way necessarily present in the consciousness of the man who seeks,

as Pascal says, to transcend himself in order to be completely human, its matter comes from either inspiration or tradition, written or oral; it is given by teachings and practices whose origin more or less escapes our observation, and in certain cases it also may be called supernatural.

In reality, this matter is more or less one with form. The more exactly it corresponds to form, the more it is deserving of respect and beneficially efficacious. It is modified, either in the meaning attributed to it, or in the elements of which it is made up, in so far as such modifications seem necessary to keep it in harmony with the sum total of human knowledge as also with pure religious feeling.

Religion is spirit, living spirit, engaged in transforming the entire world. The forms and practices which this spirit approves and adopts are those that enable it to accomplish its task.

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THE GENESIS OF CONSCIOUSNESS.

IT is often forgotten that the basis on which modern theories rest is narrower than that of their predecessors. Adam Smith did not think of himself as an economist but as a philosopher. So did Locke and Hume. Even John Stuart Mill spent as much time on psychology as he did on economic discussions. His *Political Economy* was an accident due to his failure to find a basis for a study of character. Utilitarianism, associational psychology, classical economics, and English philosophy have the same roots and the same background. The separation came through the failure of these disciplines to meet the conditions set by evolutionary theories. They went down to be revived again under new names by men who no longer thought of themselves as philosophers but as workers in science. Yet the new is merely the old order under modern titles. William James says pragmatism is a new name for an old philosophy. He is right, for Locke and Adam Smith were pragmatists even if they did not have the word.

To-day we think of behavior psychology as something new; yet it is merely a clearer statement of problems which earlier writers tried to solve by cruder methods. There is little difference in saying with Locke that the mind is blank, and in saying with the behavior school of psychologists that the content of consciousness should be ignored. Both schools agree that the sources of action are outside of consciousness. Both view human nature as the behavior of men seen in their daily life. This has become the eco-

nomic interpretation of history if measured in its effect on masses and epochs. It is behavior psychology if tested by the reactions of individuals. It is Freudian psychology if sex impulse is emphasized. It is biology if human nature is broadened enough to include problems of heredity. All these sources make contributions to the new view, and only by their revaluation on some common ground can the unity be restored which gave force to the philosophy of the last century.

The older psychology was a soul philosophy in whose terms all was explained. In the transition from it to nerve psychology it was assumed that the nerve did what the soul was supposed to do and in a similar manner. For every soul function a nerve was sought which performed mechanically what the soul was thought to do. Old concepts were thus retained under new names which could not stand an examination on their own ground. Psychology can free itself from the resulting confusion only by ceasing its endeavor to make nerve action correspond to the predicates of soul supremacy and by putting in their place known facts as exemplified in bodily activity. We should study nerves in their objective manifestation as behavior before we try to observe their effects on consciousness. What the body nerves do brain nerves also do but nothing more. If there is a discrepancy between these known powers and those manifested in our conscious life we should attribute the difference to some other force or to some complexity not yet understood. This method does not invalidate either what we know of nerves or of consciousness. It merely warns us against hasty identification and slipshod methods of passing from one to the other.

To understand our bodies we should start from the simple functions of assimilation, circulation, and muscular activity. These form a kinetic system composed of autonomous organs which generates energy. By it food is ac-

quired, digested, circulated, and discharged. In contrast to it is a system of control by which external adjustment is wrought. The latter is the central nervous system through whose development life is preserved and activity encouraged. The nerve is not the source of energy, nor does it have any but a subordinate place in the vital organs upon which life depends. We should start, therefore, not with a study of nerves but with an examination of the autonomic system which it controls but does not make. Digestion, circulation, and muscular activity still take place according to methods older than the nervous system. We must shut from view the action of the central nervous system to see what the kinetic system can do and how it does it. Its movements are always of a tropic nature. Heat, light, and gravitation are the first controllers of action, and the type of action is either that of advance or retreat. Next in order is the influence of surplus and deficit. As soon as organisms can store energy they can be aroused to activity by internal stimuli, and with further development these internal states become more effective than the external ones resulting from light and heat.

At a higher level we find organisms with a better developed muscular and circulatory system. Their striking characteristic is the symmetrical development of opposing parts. We can almost say of the higher types that they are two organisms, each side being practically a duplicate of the other. The result is rhythmic action, each part ceasing to act while its opposite discharges its energy. This rhythmic action is not different from the tropic action of simple organisms. It is merely excited in a different way. The animal has lost the control exercised by external agents such as heat and light, and their place is supplied by the states of the blood. When the blood was cold, a mere reflection of external temperature, external forces aroused activity. Now hot blood can stimulate activity

and thus energize parts by its presence or check action by its absence. Surplus and deficit thus become the determiners of action. Blood flows to one side, accumulates a surplus and discharges; while it is inactive the other side functions in a like manner. When the central nervous system begins to play its part it alters the flow of blood and checks the rhythmic action characteristic of the kinetic system.

If we judge animals by their behavior, evidence of the existence of a central nervous system is shown in persistent action and in attention. The rhythmic action of the kinetic system is changed into monistic action. Movements are continued even when they are painful. On the other hand, parts with a surplus have their activity checked with a resulting accumulation of surplus which vainly endeavors to discharge itself. The net result of this abnormal situation—parts pained by a deficit and parts overloaded with a repressed surplus—is a better adjustment due to nerve control; but we should not forget the cost in the abnormalities it creates. To this struggle our emotions are due. Consciousness is an attempt of the ultimate life forces to escape domination and not any evolution of their master, the central nervous system. If a break can be made from a lower to a higher realm the dominance of the nerve can be replaced by the dominance of the blood, through which comes a reinstatement of the ultimate life forces in their original position of power. The nerve becomes a servant, not a master. Adjustment yields to the sweep of inborn activities. Our goal is no longer to attain the objective but to voice the subjective.

This seems an impossible task with the tools at hand, yet if we look in the right quarter the road is simplicity itself. The characteristic expression of nerve action is persistence and attention. Measure these in terms of muscle and we have strain. Parts are kept active after their

supply of energy is so low as to cause pain. Strain also starts the activity of adjacent muscles by an overflow of energy. Nerve action overdoes itself by exciting more movement than is needed for the desired effect. If we raise the leg we also set the jaw and swing the arm. A series of overflow movements is excited by each nervous strain. Take off the strain and a release of energy follows in the repressed parts. Thus strains and releases are the new alternates, each in turn affecting not one part but many parts. One group of movements is induced by strains and another by the release of suppressed energy which follows the strain.

With these facts as a background the problems of consciousness can be faced. Consciousness is an illumination, a transformation of energy from motion to light. Energy and light have the same antecedents, and from one to the other transformations can be made. Consciousness starts with an overflow of energy due to strain meeting some resistance which intensifies its effect. An electrical current transforms energy into light when it is forced to pass over a slender wire. If we repress bodily energy and let it escape under restricted conditions we produce organically what the electric current does mechanically. This is what strain, repression, and release do if their relation is such as I have described. Nervous control causes an overflow of energy and its repression at other points. The release and escape transform energy into an illumination. Consciousness in its earliest form is thus an overflow of energy due to strain and a release of energy which follows strain. Either of these would produce the desired transformation of muscular energy into light while both together would tend to make the illumination permanent. Such a transformation may be readily observed in the case of fireflies. The movement of the wings is accompanied by the radiation of light. The escape of energy in this case produces an

external effect. Change the escaping flow from the body to the brain and an elementary form of consciousness would result.

I want to repeat this thought so essential to my position. Can an energy strain be transformed into a color scale? Is the background of consciousness a change of color and shade, or does a series of ideas flow through consciousness in an immaterial chain? When we repress ideas, do we shove some entity into a subconscious cavern or do we extinguish the color form by which they are recognized? As we decide these questions we determine the form of psychology we accept, and also determine the relative parts which blood and nerve play in the formation of ideas. The problem is: Will a strain produce a mental illumination causing it to be recognized in consciousness not as effort but as form? Do we see something when we strain, and if so, what? My reply would be that blood pulses create the illumination which underlies consciousness and that they are intensified by any strain directing energy into particular channels and repressing it in others. The same localization of energy occurs in the physical background of consciousness as occurs in the strained part of the body. Parts of the field of consciousness are darkened by a deficit of blood and other parts have an increased illumination due to augmented blood pressure. The field of consciousness thus presents a figure which is light or dark, simple or complex, wavy or fixed, as the underlying strains change in their intensity. We thus have a color scale which reflects bodily conditions and through which they may be interpreted.

When in the dark I lie with my eyes closed the changing blood pressure often creates distinct figures. If these slight alterations in pressure are capable of an interpretation it is easy to see how bodily strains can produce more intense effects. Strong contractions in one muscle certainly create

an overflow of energy which contracts other muscles. If so, it must derange the circulation in other parts of the body and thus affect the blood pulses of the brain. The rhythmic contraction of related parts causing this condition makes it possible to judge of bodily stresses by the resulting mental picture.

An illumination of this sort would have no adjustive value. It would be merely an escape of surplus energy as is the illumination due to the movement of a firefly. But we should remember that conscious illumination is complex due to the relation of strain and release. It is, therefore, never complete. Some parts of the field of consciousness are illuminated by the release, while other parts are darkened by the strain. Each specific content of consciousness has for this reason a form and these forms are capable of an interpretation. By these forms activity can be judged, and through them we gain the basis of our logical judgments.

The dominance of strain gives one form, the dominance of release gives another. We thus have two forms with every nervous restraint; A as it produces strain, and B in the subsequent release. If the net result of strain and release is organically favorable, we identify A and B and think of the two as interchangeable. A is B and B is A. This is the logical basis of substitution, and it happens wherever A or B are the index of favorable organic effects. But when the effect of strain and release is organically antagonistic we set A and B over against each other and call them opposites. From this tendency we get the basis of our moral judgments. We then pronounce the action which caused the strain to be bad, or we seek to restore harmony by some suppression of the release. In either case we acquire the concept of the good and bad and build emotions which help us gain the good and remove the bad. Love and hate rise in consciousness as soon as antagonism

due to strain and release appear. From them comes the concept of free association due to the repeated substitution of similars and the concept of antagonism upon which elimination depends. Hence arise the three elements of conscious thought — illumination, form, and interpretation. With their aid the gap between the physical and the mental can be spanned.

To understand this process we must realize how color and form are interpreted in the ordinary experience of men. The illumination varies from an almost complete field of bright color to a repression leaving only dark lines visible in consciousness. Strain always exerts some influence and thus the illumination is not complete. The nearest to it is the presence of lines or spots in the illuminated field, and then the associated concept is that of a face. The man in the moon is an example of primitive association. Increase the suppression of light, and at the other end of the scale we have wavy lines which are interpreted as snakes and other loathsome creatures. These variations I shall call the chromatic scale. From the resulting combinations all initial interpretations are derived. At the top of the scale, where the illumination is full, the association is with the sun, the creator, the hero, and those concepts out of which arises our ideal of perfection. Less complete, the association is sexual. Any intense figure enclosing dark spots or lines brings to the woman the concept of a man and to the man a like concept of a woman. Still lower on the scale, where the edge becomes wavy, we seem either to be moving through the air and then the bird concept arises, or something comes to take us away, and then we think of a horse. If the illumination is still more obscure we seem unable to move, and the form is that of a bear; while still lower, being conscious only of wavy lines, we are filled with horror at our helplessness and interpret moving objects as hideous reptiles.

These figures in turn arouse tropic movements. An intense light excites a forward movement while a dim light provokes a retreat. The bright we call the good; the dark excites fear; while wavy lines are interpreted as movement. All the primary impulses can thus be excited and an interpretation made in terms of the external world. We read the scale and act on it instead of waiting for the direct action of the external world on our bodily organs. Our movements are excited no longer by heat and cold but by forms projected into consciousness by the chromatic scale. Sense-perceptions are reactions which throw color images into consciousness and then move muscles in harmony with the interpreted forms. Color and form are intermediates between two systems each of which were once independent in their action. It is thus that a few primary impulses get organized so that they perform the functions demanded by a complex adjustment Not a single function is added. The mechanisms of simpler organisms merely show themselves in new combinations.

Such a picture, however satisfactory, does not account for the thought-processes of men as contrasted with their impulsive reactions. There is as great a need of control over visual impulses as over bodily movements. This need the central nervous system supplies by the same means as it controls bodily movements. If persistence is the measure of nervous control of bodily action, attention is its mark in conscious acts. Attention is a mental strain of the same nature as bodily strains. It consists in keeping the flow of thought confined to a single object when, if controlled by impulse, thought would be a series of substitutions. Each thought would then be interpreted in terms of some other thought. This new thought would be put in place of its predecessor, to be displaced in turn by that concept through which it is interpreted. Such a series is a free association, the most familiar form of which

is a dream. Attention disturbs this flow of free association and puts in its place the fixed series which we call the logical. To be logical, thought must flow in a fixed groove created by the strain of attention. Grooved thought is the series which leads to survival and hence we call it the truth. True thoughts thus correspond to the flow of objective impressions and lead to the repression of the internal flow of free associations. The believer in truth rejects as false all this internal flow of free associations, shuts it out of his consciousness and builds his scheme of thought only on impressions received from the external world. He does this by means of repressions through which the natural sequence of free association is prevented. There are also groups of compulsions, by which I mean that with given ideas other ideas arise in consciousness and retain a fixed order. These suppressions and compulsions are never complete. The actual thought of man is a combination of antagonistic elements each of which in striving for the mastery forces its opponent into distorted forms. The opposition we cannot prevent, but the bad effects might be obviated by a better understanding of the process by which it is promoted.

This analysis of thought appears strange, but it is not so different from the traditional analysis as at first sight may seem. The fundamental contrast is new, but, when made, the secondary divisions can be readily made to correspond to those now in use. In its origin thought is partly internal and partly external. Internal thought is a process, a constant shifting from one concept to another, the underlying principle being that of substitution. External thought is compulsive. It may fade, but it never changes. When a concept returns it has exactly the same form as before and the same accompaniments. These are impressments if they arise from the action of the senses, or compulsions if they result from strains due to the antecedent action

of the nervous system. Impressments and compulsions are unchangeable elements which we can analyze but never transform. They become parts of thought-processes and thus create thought movement only as we substitute for them some concept of internal origin. Through this substitution chains of reasoning are started which flow along a natural channel unless some nervous repression alters their course. Natural thought is a shifting from concept to concept, the new being a substitute for the old. Reasoning of this sort has as premises either impressments or compulsions; then begins a series of substitutions out of which we draw a conclusion. The intermediate steps we may change, but the original premise stands firmly above criticism because it is an impressment or a compulsion and thus beyond the power of conscious thought to alter. When tested by their origins we find that the impressments stand but the compulsions crumble. They are neither a reflection of nature nor a part of our heredity. When they are excluded, thought rests exclusively on sense impressments and on the free associations due to the process of substitution. Impressions are unalterable and uncombinable. The unity we give them comes from no affinity they have for each other, but through the mental compulsions which have their origin in strains. In our free thought-processes impressments enter only through substitutions. They are thus transformed from isolated facts into the mass-formations which free association is capable of making. Attention, which is nervous, analyzes and isolates. Association compounds. These are our only thought-processes; through them thought sinks to mathematical points or rises to mass infinity. The one is the dominant tendency in science; the other gives zest to the flights of the imagination.

Such is the process of thought in its primary terms. To make it correspond to the full consciousness of adult life we must show its application to the combinations of

ideas we call memory. As an element in consciousness, memory is a series of associated ideas which differs from free associations in that the order is fixed. Free association is a shifting from one vivid idea to another, the basis of the shifting being the yearning for wish-fulfilment. Memory does not follow this order because the flow of thought is cramped by repressions which force it to move in fixed grooves. Thoughts which hinder adjustment are repressed, while those are intensified which reflect external conditions. Memory is thought moving through or controlled by nervous tracts and reactions. The thought which follows each presented concept is that which excites a nervous reaction and not that which the process of free association would provoke. Memory is thus a series of nervous reactions and not a series of free associations. The nervous reaction is the connecting link between ideas and not the rhythm of blood and muscle which lies back of free associations.

This analysis is readily transformed into the terms of psychology. Writers of long ago discovered that associations were resolvable into three groups, contiguity, similarity, and contrast. This grouping suffices for a mere classification but is no aid in the search for origins. The antecedents of contiguous associations are nervous. The contiguous is that which arises through the nervous reaction which an object excites. Contiguous associations are a series of impressments and not a series of substitutions. Such thought is mechanical but not rational. It is only the two other types of association, similarity and contrast, which are in any true sense associations. They form a process which takes place within consciousness and each is necessary for the other. Substitution acts through the blood and muscular rhythm it excites, and not through the nervous reaction. If we had no related organs which reciprocally act we could have no process of substitution

and hence no way of arriving at the thought of similarity and contrast. Association is the interpretation of one object through some other which follows it in the rhythm of activity and replacement. There is a strain, a discharge, and then a rest during which time the activity is transferred to the reciprocal part. Two organs thus take a part in each substitution, the repetition of which creates the series of free associations. We see an object, we think of its name which is its recognition as a sound; then the associated sound is reflected back into the field of vision and there arises in a form related to the sound which was its antecedent. Each series of associated ideas takes a definite path fixed by the associations which arise with each new transfer. Word associations, as the process continues, grow in clearness while visual associations grow more intense. We thus attain a union of clearness and vividness which no process of impressment could attain. The order of these ideas is not that of the external world nor that imposed by the nerves. Free associations move toward a goal set by internal impulse and not toward that imposed by our nervous reactions.

The nerves thus play the same part in thought as they do in bodily control. No one assumes that bodily energy comes from the nerves. It is plainly a rhythmic flow exciting in turn each of two symmetrical parts. Any sudden shock, the stubbing of a toe, a sound, a sight, or the feeling of pain starts a nervous reaction which injects a foreign element in the series and forces action into a new channel. All shocks are injects which disturb rhythmic movement of symmetrical parts. This makes the difference between the rhythmic flow of energy induced by the muscular activity, and the reactions which the nervous system is capable of exciting. These nervous injects are the ultimate premises from which we reason as to ends. The nervous inject is a substitution of a higher value than the thought

which its rise suppresses. Let us assume that in a natural series the conclusion is reached that the eating of pork is injurious. If instead of saying this it is affirmed that God prohibits the use of pork, the concept of God is substituted for the evidence about pork-eating. By this means antagonism to pork-eating gains a vividness which the injury of pork-eating could not evoke. This vivid concept of God is a nervous inject which we must explain through some antecedent shock which suppresses the natural flow of thought. All moral laws arise in this way. They substitute nervous shocks of high adjustive value for the crude flow of internal thought. Reality thinking is thus an injection of a nervous reaction as initial point of a new thought-series. When reasoning deductively we say, "It is either A or B." We do not decide in favor of A by means of direct evidence. We say, "It is A because it is not B." It is thus the aversion to B and not the proof of A which determines the decision. The aversion to B is a nervous inject due to antecedent experience. It represents a repression the source of which we may be unconscious of but which has been repeated often enough to be the cause of a nervous reaction. The repressed, however, is not what we hate but what we love. It is a wish, a yearning, reflecting our natural self. But when its presence excites a nervous reaction we substitute for our emotion of love the acuter pain which the nervous reaction evokes. A love is thus changed into a phobia, and the censor instead of being avoided has its power strengthened.

Here is an example from personal experience. I was discussing with a friend the utility of certain beliefs. He was much wrought up at my position and finally expressed his emotion in this form: "If I believed as you do and had only one year to live I would indulge in every sensation that Broadway affords, getting the most possible out of the year." This was put forward as a convincing argument;

but it revealed more of his interior than he was aware. A description of his family will show the content of his inner self. His father was known to have broken every commandment but the eighth, and his brother lived so fast that he died before thirty. His mother was a saintly woman, a lady bountiful who put many an ill-gotten dollar to good use. My friend had an intense love for his mother and accepted all her compulsions as divine commands. His life was as pure as hers, but he had his father's heredity. This was voiced in his denunciation for all his father did, but this denunciation was also an outlet for the pent-up emotions which he inherited but never gratified. It is plain that the intense expressions he used to me were the reflection of his day-dreaming. He would like to be bad but dared not. His expressed thought was thus turned into a form which strengthened the censor instead of thwarting it. The nervous complexes formed in this way lead to superstition through their vagueness and to dogmatism through their cohesion. The purification of thought does not come through them but through free associations. We thus reach the source of mental activity by a route which makes no demand for elements not found in the simple organisms. The blood pulses are strong but fickle. The nerve mechanisms are steady and coordinated. The net result is a series of revolutions due to temporary pulses outwitted by the better organized nerves. Adjustment thus dominates over internal impulses creating an approach to rational conduct.

With this analysis of the elements of consciousness as a basis the new thought can be readily related to older philosophies. The limitations placed on the sensory functions make the part they play in the formation of ideas resemble that given to the sensory powers by the transcendental philosophers. The senses can present; the central nervous system can control; but beyond this neither can

go. Some other course must be sought for the processes by which thought can rise to the level we know it is capable of attaining. In so much the transcendentalists were right. They could challenge their opponents' position and win a negative victory. But on the positive side they failed because they sought to find for thought some super-sensory origin instead of looking for a pre-sensory origin. The from-within elements of consciousness are older than the from-without. Life was blind before the rise of the nervous system, but though subordinated its ultimates still persist. The path of progress consists in freeing the elemental life pulses from the distortion which ages of adjustment have imposed. The within must impress itself on the without instead of the without distorting and suppressing what it has not the power to change.

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CRITICISMS AND DISCUSSIONS.

THE PRINCIPLE OF PARSIMONY AND ETHICAL NEUTRALITY.

My object in writing this short note on Professor Laird's extremely interesting paper, published elsewhere in this number, is to deny that certain unpleasant consequences follow from Mr. Russell's acceptance of "Ockham's razor" as the supreme maxim of scientific philosophizing.

Professor Laird believes, for example, that Mr. Russell involves him in a falling away from the "ethical neutrality" he has claimed for his philosophy. For Mr. Russell's entities are not simpler in all respects, writes Professor Laird; and if it be claimed that they are simpler in *important* respects then a subjective standard is introduced. We must here disentangle two separate questions.

1. It may be urged in the first place that Mr. Russell abandons his "supreme maxim" as soon as he accepts a large number of entities—by which, I take it, is meant the *sensibilia* out of which "things" are constructed. Now it does not seem to me that this view could be held if it were once realized what logical atomists are trying to do. Faced by a chair, the atomist—seeking to eliminate all unjustifiable entities—resolves it into a complex of *sensibilia*. He may thereby be led to express in a very intricate manner what is simple to common sense, just as the logistician expresses $2+2=4$ in a garb highly unfamiliar to the beginner in arithmetic. But the atomist is nevertheless in this process strictly adhering to the principle of parsimony in paring away every kind of entity of which he has no direct awareness. The scientist, on the other hand, proceeds in the opposite direction: he multiplies entities unceasingly.

The chair becomes a complex of molecules, which becomes a complex of atoms, which is in turn resolved into a tangle of electrons—and so on. The ultimate goal of this multiplication of entities is the construction of a conceptual scheme which is economically "descriptive" of the increasingly complicated perceptual field opened up by experiment. Professor Laird's argument that the sole aim of science cannot be descriptive because models are widely used, seems to me to miss the whole point about models. For, in the hands of a Maxwell, models are used precisely because they enable those concepts to be formulated between which simple relations hold—those relations having the property that the conceptual scheme so set up expresses the empirical laws in the corresponding perceptual domain. We may conclude, then, that while science is ready, for the purposes of economical description, to multiply entities philosophy seeks to limit such entities (even with loss of economy in statement) to those of which we are directly aware. In other words, the scientific principle of economy *is* anthropomorphic; the principle of parsimony is not.

2. It may now be urged, however, that though Mr. Russell's entities are simpler than those of science, as being all of one type, yet our choice of this system is egocentric. This seems to be what is at the back of Professor Laird's contention. In this he is adopting a similar attitude to that of Dr. Schiller who urges that the principle of parsimony is a maxim of practical convenience. "To a non-human mind," he says, "that was not pressed for time but disposed of all eternity it would be unmeaning or repugnant."[1] This argument depends for its plausibility on the different forms in which the "razor" is expressed. If, however, we express it as *Pluralitas non est ponenda sine necessitatem,* then we merely assert that you must not suppose that more things exist than you have evidence for. The criterion is the appeal to the external world. The standard is therefore not subjective, unless it is assumed that everything, including sense-data, is subjective. Furthermore, from what has been said above it is clear that this is *not* a maxim of "practical convenience." It is, in fact, usually convenient to contravene it. If, for example, we always used Mr. Russell's constructions instead of "things," our novels could only be read by those who disposed of all eternity. And if we avoided the use of the entity "number," Jones minor's text-book of arithmetic would be

[1] *Mind,* N. S., Vol. XXIV, p. 402.

weightier than Jones minor. In other words, in practical affairs we emphatically do not (and never shall) use the razor to cut away our tables and chairs and numbers and electrons. Ockham's razor is on the contrary an impractical maxim only applied in those moments of critical analysis when we desire to know what errors we are daily committing from pragmatic motives.

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THE LOGICAL SIGNIFICANCE OF "OCKHAM'S RAZOR."

In adopting the principle of parsimony or "Ockham's razor" as the supreme maxim of methodology, Mr. Russell seems to have been guided by esthetic motives. To some of us, indeed, it seems to be more beautiful to start the development of a science in deductive form from three fundamental or "primitive" ideas instead of from four and to define all the rest in terms of these three, and even more beautiful to start from two primitive ideas. But there seems to be a *logical* basis for this preference, for the reduction in the number of primitive ideas implies a discovery of dependence between those ideas at first taken to be indefinable. I shall try to make this more precise.

Formally, the ideas of a deductive science, both the primitive ones and the ones that are defined in terms of them, may be regarded as the "unknowns" in an algebraic problem, between which equations, representing the definitions, subsist. This is at the basis of the algebra of logic. If the number of independent equations is equal to that of the unknowns, the unknowns can be wholly defined. This is the case with the science of arithmetic, according to modern ideas; for number and the other ideas with which arithmetic is concerned are all definable in terms of logic. But if the number of independent equations is less than that of the unknowns, some of the unknowns are indefinable, and in fact it may be more convenient to leave more of the unknowns undefined than are strictly necessary, and preserve the equations between these indefinables. In algebra we may find that, with the unknowns x, y, z, we know that $x=f(z)$ and $z=\phi(y)$, and consequently that y is strictly speaking the only indefinable; but it may be symbolically impossible

to *express* x as a function of y. Of course, we may have an indefinite number of defined ideas, and it is easy to see that this number does not affect the question as to how many indefinables there are.

For example, Whitehead and Russell, in their theory of deduction, have expressed *implication* and *joint assertion* of propositions in terms of *negation* and *disjunction*. Sheffer and Nicod have succeeded in defining all the above four ideas in terms of a new function of propositions p and q denoted by "p/q" and which can be defined in the system of Whitehead and Russell as "Either p or q is false." Thus, in the system of Whitehead and Russell we have five ideas of which two are indefinable, while in Sheffer's system we have five ideas of which one is indefinable. The latter is preferable because it discovers a relation between the two undefined ideas of Whitehead and Russell; though this relation does not appear capable, in our system of symbols, of being expressed without making use of Sheffer's indefinable. Thus the principle of parsimony appears, from a logical point of view, to be simply the maxim that logical analysis is to be carried as far as possible; and this is no more than Dedekind's maxim that what *can* be proved *is* to be proved.

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INDEPENDENCE PROOFS AND THE THEORY OF IMPLICATION.

In the January number Mr. Lenzen writes a criticism of the traditional symbolic logic as exemplified by the system of Whitehead and Russell's *Principia Mathematica*. His criticism is that it fails to give a correct theory of mathematical deduction. The attempt is made to prove that, in certain cases, of two mathematically independent postulates one implies the other according to the method of the *Principia*.

The present writer was, at first, in sympathy with the criticism; but finally the error involved was detected. The chief illustration is taken from two independent postulates for algebra:

$$\text{A2} \qquad (a+b)+c=a+(b+c),$$

$$\text{M5} \qquad a\times b=b\times a.$$

Applying the symbolic methods,

$$M5 \cdot c \cdot A2 \, c \, M5.$$

But, says the critic, M5 is true and therefore by the Principle of Inference (any proposition implied by a true proposition is true),

$$A2 \cdot c \cdot M5.$$

The error of the criticism lies in asserting the truth of M5. The Principle of Inference requires M5 to be true universally; Huntington's proof of the independence of A2 and M5 involves, however, finding at least one case or system in which M5 is *not* true and in which A2 is true. Or we may put the matter in another form. If you once assert as true the postulates you are considering, you can no longer investigate their independence; because you cannot find a case in which one is false. On the other hand, if one is false, in some system, it is no longer true universally, and the Principle of Inference used by Mr. Lenzen has no support.

In fact, however, a deductive relation necessitates the use of at least one formal implication, which is a relation between propositional functions. Now it is, perhaps, a fair criticism of the *Principia* that it does not specify the various kinds of variable in terms of which a propositional function may be expressed. It may be a mathematical variable such as x or the shape of a triangle, or it may be a proposition, or it may be a situation or system in which it is asserted. This last view is unfortunately not mentioned by Whitehead and Russell, though the writer believes they would agree with it.

Let us return to the illustration given before. Call the system of algebra corresponding to Huntington's postulate, S_1. A2 and M5 are asserted for the System S_1, and we may call them

$$\begin{array}{ll} A(S) & (a+b)+c=a+(b+c), \\ M(S) & a \times b = b \times a, \end{array}$$

both asserted in the system S_1.

By definition A(S) formally implies M(S) only when

$$(S) : A(S) \cdot c \cdot M(S);$$

that is:

$$(S) : \sim A(S) \cdot v \cdot M(S).$$

But Huntington's analysis shows that, for some system, say S_2,

$$A(S_2) \cdot \wedge \cdot \sim M(S_2),$$

or

$$\sim [\sim A(S_2) \cdot v \cdot M(S_2)].$$

That is to say, for this S_2, $A(S_2)$ does not imply $M(S_2)$. The formal implication falls to the ground.

Further on Mr. Lenzen says that a presumption is created that independence applies to propositions as well as to postulates. This presumption is probably an error, for any method of deduction involves the consideration of the particular propositions as special cases of propositional functions. If deductive relations could be found between propositions, these relations would also hold between the corresponding postulates, or else the propositions would not be typical cases. Contrary to Mr. Lenzen's assertion, it is possible sometimes to find deductive relations between particular propositions which correspond to independent postulates, if non-typical cases are chosen. The writer has found several but will choose one for the sake of illustration.

Consider the postulates

$$(A) \qquad a \mathrel{p} b = b \mathrel{p} a$$
$$(B) \quad a \mathrel{p} (b \mathrel{p} c) = (a \mathrel{p} b) \mathrel{p} c$$

These are independent, for if $a \mathrel{p} b$ means $(a+b)/2$, A is true while B is not; if $a \mathrel{p} b$ means a, B is true while A is not; if $a \mathrel{p} b = a\,b$, both are true.

Consider a particular case of B, namely,

$$a \mathrel{p} (b \mathrel{p} a) = (a \mathrel{p} b) \mathrel{p} a.$$

This can be deduced from (A) for

$$\begin{aligned} a \mathrel{p} (b \mathrel{p} a) &= a \mathrel{p} (a \mathrel{p} b) \\ &= (a \mathrel{p} b) \mathrel{p} a, \end{aligned}$$

using A twice.

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CAUSE AND EFFECT.

I.

The latest book by Mr. Bertrand Russell (*Mysticism and Logic, and Other Essays*, London and New York: Longmans, Green and Co., 1918; pp. viii, 234; price 7s. 6d. net) consists of reprints of ten previously (1901-1915) published papers which form a more or less complete whole and to which are added one or two notes written in 1917. The first essay, on "Mysticism and Logic," appeared in

The Hibbert Journal for July, 1914, and contains an attempt at a characterization of the scientific and the mystical impulses. Much of this essay is familiar to most of us from Mr. Russell's book on *Our Knowledge of the External World* (Chicago and London, 1914, cf. especially pp. 1-30, 45-47, 63-64, 95). "The Place of Science in a Liberal Education" appeared in *The New Statesman* for 1913 and is concerned especially with the intrinsic value of a scientific habit of mind in forming our outlook on the world. "A Free Man's Worship" and "The Study of Mathematics" are reprinted from Mr. Russell's *Philosophical Essays* (London and New York, 1910), now out of print, where they had been reproduced from magazine articles of 1903 and 1907. The first of these two essays is supplemented by a note in the Preface (p. v) drawing attention to an alteration in Mr. Russell's ethical views. The essay on "Mathematics and the Metaphysicians" appeared in *The International Monthly* for 1901 under the title "Recent Work in the Philosophy of Mathematics" (cf. *The Monist*, Vol. XXII, 1912, pp. 149-158). It is interesting to read that the tone of this essay is partly explained by the fact that the editor begged Mr. Russell to make the article "as romantic as possible" (p. vi). This reprint is very useful since the original seems difficult to obtain, and the amusing and slightly frivolous remarks about mathematics have been so often taken literally and thus misunderstood by superficial and serious people. The "Herbert Spencer Lecture" of 1914 "On Scientific Method in Philosophy" was first published in that year, and here again there is not much that will be new to those who have read Mr. Russell's larger book mentioned above. "The Ultimate Constituents of Matter" was published in *The Monist* for July, 1915; "The Relation of Sense-Data to Physics" was published in *Scientia* for 1914; and "Knowledge by Acquaintance and Knowledge by Description" was published in the *Proceedings of the Aristotelian Society* for 1910-11. It will be found that most of the views in these essays are either repetitions or developments of the views given in the above book or in Mr. Russell's *Problems of Philosophy*, first published in 1912 in London and New York.

The only remaining essay is "On the Notion of Cause," which was published in the *Proceedings of the Aristotelian Society* for 1912-13, and also, though this is not stated by Mr. Russell, in *Scientia*. This essay contains some things which do not appear in either of the books just mentioned, but most of those that do not

so appear go back to Mr. Russell's out-of-print *Principles of Mathematics* of 1903. It seems to me that this essay suffers from the fact that the view that the concept of *cause* can be replaced by the mathematical concept of *function* has been almost a commonplace of science since Mach wrote,—say between 1866 and 1883,—and that Mach's name is not mentioned. The essay is thus summed up (pp. 207-208): "We found first that the law of causality, as usually stated by philosophers, is false, and is not employed in science. We then considered the nature of scientific laws, and found that, instead of stating that one event A is always followed by another event B, they stated functional relations between certain events at certain times, which we called 'determinants,' and other events at earlier or later times or at the same time. We were unable to find any *a priori* category: the existence of scientific laws appeared as a purely empirical fact, not necessarily universal, except in a trivial and scientifically useless form. We found that a system with one set of determinants may very likely have other sets of a quite different kind; that, for example, a mechanically determined system may be teleologically or volitionally determined. Finally we considered the problem of free will: here we found that the reasons for supposing volitions to be determined are strong but not conclusive, and we decided that even if volitions are mechanically determined, that is no reason for denying freedom in the sense revealed by introspection, or for supposing that mechanical events are not determined by volitions. The problem of free will *versus* determinism is therefore, if we were right, mainly illusory, but in part not yet capable of being decisively solved."

Just as in the *Principles,* Mr. Russell considers the case of a swarm of particles in abstract dynamics, where the only thing which, it seems, might correspond to what is called "causality" is the fact that, since the whole path of the swarm is determined by a system of certain ordinary differential equations of the second order, all the arbitrary constants are fixed if we know the configurations of the swarm at any two given times, and thus the configuration at any other instant whatever is uniquely determined by the above differential equations and the above two fixed configurations. Mr. Russell does not state this well-known fact clearly, and thus the uninitiated are left dumb with admiration at what seems mysterious penetration of a very brilliant man. If Mr. Russell had put the matter frankly, he would certainly have committed

what would doubtless seem to him a double crime of stating a technical platitude, but still it does not seem that it would have escaped him or others that, in this discussion, the question as to what meaning can be given to "causality" is implicitly limited to swarms whose paths are defined by ordinary differential equations of the second order, so that there are unanalyzed assumptions as to the nature of the functions which give the dependences of the co-ordinates on the time which are fundamental, and that the grounds for generalization to other physical "swarms" ought to be given.

The logical analysis (pp. 180-192) of what philosophers, according to the definitions they have given, have professed to understand by the word "cause" and allied words is often very amusing. Still this analysis is based on principles which are quite well known to modern logicians; for example, a discussion (such as that on p. 187) of such statements as that the effect follows the cause "immediately" can, of course, quite easily reduce the statement to nonsense by the remark that there are no such things as "consecutive points" in space or time. Then we come to the really important part of the article: "In spite of these difficulties, it must, of course, be admitted that many fairly dependable regularities of sequence occur in daily life. It is these regularities that have suggested the supposed law of causality.... I do not deny that the observation of such regularities, even when they are not without exceptions, is useful in the infancy of a science.... What I deny is that science assumes the existence of invariable uniformities of sequence of this kind, or that it aims at discovering them.... In short, every advance in a science takes us farther away from the crude uniformities which are first observed, into greater differentiation of antecedent and consequent, and into a continually wider circle of antecedents recognized as relevant" (pp. 187-188). Again, "such laws of probable sequence, though useful in daily life and in the infancy of a science, tend to be displaced by quite different laws as soon as a science is successful" (p. 194).[1] Thus, in the motions considered in an advanced science such as gravitational astronomy, "there is nothing that can be called a cause and nothing that can be called an effect; there is merely a formula. Certain differential equations can be found, which hold at every instant for every particle of the system, and which, given the configuration and velocities at one instant, or the configurations at two instants, render the

[1] Cf. *Ext. World*, p. 220.

configuration at any other earlier or later instant theoretically calculable. That is to say, the configuration at any instant is a function of that instant and the configurations at two [fixed instants]. This statement holds throughout physics and not only in the special case of gravitation" (p. 194). This, then, brings us back to the point of view of the *Principles*.

Then Mr. Russell proceeds with the replacement of the old "law of causality" by the idea of a mathematical "function": "No doubt the reason why the old 'law of causality' has so long continued to pervade the books of philosophers is simply that the idea of a function is unfamiliar to most of them, and therefore they seek an unduly simplified statement. There is no question of repetitions of the 'same' cause producing the 'same' effect; it is not in any sameness of causes and effects that the constancy of scientific law consists, but in sameness of relations. And even 'sameness of relations' is too simple a phrase; 'sameness of differential equations' is the only correct phrase" (pp. 194-195).[2] "If the 'law of causality' is to be something actually discoverable in the practice of science, the above proposition has a better right to the name than any 'law of causality' to be found in the books of philosophers.... No one can pretend that the above principle is *a priori* or self-evident or a 'necessity of thought.' Nor is it, in any sense, a premise of science: it is an empirical generalization from a number of laws which are themselves empirical generalizations" (p. 195).

The old "law of causality" is not assumed by science, but "something which we may call the 'uniformity of nature' is assumed, or rather is accepted on inductive grounds. The uniformity of nature does not assert the trivial principle 'same cause, same effect,' but the principle of the permanence of laws. That is to say, when a law exhibiting, e. g., an acceleration as a function of the configuration has been found to hold throughout the observable past, it is expected that it will continue to hold in the future, or that, if it does not itself hold, there is some other law, agreeing with the supposed law as regards the past, which will hold for the future. The ground of this principle is simply the inductive ground that it has been found to be true in very many instances; hence the principle cannot be considered certain, but only probable to a degree which cannot be accurately estimated" (p. 196; cf. p. 192).

[2] Cf. what seems to be a contradictory statement in *Ext. World*, p. 214.

We will now give a short account of the not dissimilar views previously published by Mach and Karl Pearson.

In papers published in 1865 and 1866, Mach (cf. *Conservation of Energy*, Chicago and London, 1911, pp. 89-90; cf. p. 95; this book is a translation of an important work published by Mach in 1872) maintained that in physics "space" and "time" are merely shorthand symbols used in the representation of the dependence of phenomena on one another. In the work of 1872 just referred to Mach replaced the concept of *cause* by the concept of *function* (cf. *Mechanics*, 3d ed., Chicago and London, 1907, p. 555, and *C. of E.*, p. 9), and also shortly expressed his fundamental standpoint in psychology, physics, and psychophysics (*C. of E.*, pp. 91, 95) as sets of inquiries into the connections among themselves and with each other of (1) our presentations, and (2) our sensations.

According to Mach, the "law of causality" is "the presupposition of the mutual dependence of phenomena" (*C. of E.*, p. 61; cf. p. 102). "The business of physical science is the reconstruction of facts in thought.... The rules which we form for these reconstructions are the laws of nature. In the conviction that such rules are possible lies the law of causality. The law of causality simply asserts that the phenomena of nature are dependent on one another. The special emphasis put on space and time in the expression of the law of causality is unnecessary, since the relations of space and time themselves implicitly express that phenomena are dependent on one another" (*M.*, p. 502). Indeed, "we have grown used to considering natural phenomena as dependent upon one another" (*C. of E.*, p. 59); and, since "temporal" and "spatial" determinations are merely, as indicated above, determinations of phenomena by means of other phenomena, we can eliminate the mention of time and space in Fechner's formulation of the law of causality: "Everywhere and at all times, if the same circumstances occur again, the same consequence occurs again; if not, not" (*ibid.*, pp. 60-61; cf. p. 98). Thus the law of causality is the supposition that between the phenomena $\alpha, \beta, \gamma, \ldots, \omega$ certain equations subsist, the number and form of which are to be found empirically (*ibid.*, pp. 61-62); but we can never discover anything which one would try to express by the phrase "the behavior of the totality of phenomena" (*ibid.*, pp. 62-63). "Let us call the totality of phenomena on which a phenomenon α can be considered as dependent *the cause* [of α]," then α is determined *uniquely* by the cause (*ibid.*, pp. 63-64; cf. *M.*,

p. 502). The principle of "sufficient reason," which was used in mechanics by Archimedes, Poinsot, and others, is, in a paper published in 1868, "only another form of the law of causality" (*ibid.*, p. 81). In 1872, it is the "inverse of" (*ibid.*, p. 65), or is "not essentially different from" (*ibid.*, p. 66) that law (*ibid.*, pp. 65-69; cf. *M.*, p. 502), and is, like that law, barren in default of positive experiences. The principle of excluded perpetual motion is another form of the law of causality: "If a group of phenomena is to become a source of continual work; this means that it shall become a source of continual variation of another group of phenomena. For, by means of the general connection of nature, all phenomena are also connected with mechanical phenomena, and therefore with the performance of work. Every source of continual variation of phenomena is a source of work, and inversely" (*C. of E.*, p. 69). Then some simple consequences of the phenomena α, β, γ,... being one-valued functions of x, y, z,..., were deduced, and the facts emphasized (1) that these theorems do not apply merely to mechanics, and (2) the barrenness of the theorems without experiences (*ibid.*, pp. 69-71; cf. also *M.*, pp. 502-504).

Thus, "the theorem of excluded perpetual motion is merely a special form of the law of causality, which law results immediately from the supposition of the dependence of phenomena on one another—a supposition which precedes every scientific investigation; and which is quite unconnected with the mechanical view of nature, but is consistent with any view if only it retains a strict rule by law" (*C. of E.*, pp. 73-74). This theorem, indeed, is reducible to the purely logical truth that, if λ, μ, ν,... are one-valued functions of α, β, γ,..., and α, β, γ,... pass to values α', β', γ',..., so that λ, μ, ν,... pass into λ', μ', ν',..., then, if the set α', β', γ',... be brought back to α, β, γ,..., the set λ', μ', ν',... will return to λ, μ, ν,... (*M.*, p. 503).

* * *

The views of Karl Pearson are to be found in his book entitled *The Grammar of Science* (2d ed., London, 1900), and seem to have been developed independently of those of Mach. Pearson vigorously protested (cf. pp. 78, 81, 112) against the use of the word "law" when speaking, for example, of gravitation, and suggested the substitution of the word "formula." This is, of course, practically the same thing as the replacement of the notion of *cause* by the conception of *function*. Pearson's work is perhaps better known

than that of Mach, but, from the point of view of theory of knowledge, it is very inferior to Mach's work. For instance, the treatment of causality and probability (pp. 128-129, 130) may be mentioned.

* * *

In what follows we will indicate the changes introduced, from 1912 onward, in Mr. Russell's views of the nature of causality,—its dependence on the "principle of induction" and so on. Then we will state some objections, which seem to me to be quite conclusive, to these later views; and finally we will give a sketch of a theory of causality as an *a priori* principle together with a suggested definition of "probability," in so far as it is not merely a way of admitting ignorance as to the validity of certain deductions from completely determined premises,—as when, for example, we say "it is probable that the proposition p implies the proposition q."

According to Mr. Russell (*Principles of Mathematics*, Cambridge, 1903, p. 478), "Causality, generally, is the principle in virtue of which, from a sufficient number of events at a sufficient number of moments, one or more events at one or more known moments can be inferred." This principle, which seems to be the same as the one which, as we have seen, Mach accepted as *a priori*, was founded, in Mr. Russell's *Problems of Philosophy* (first published in 1912, pp. 93-107), on the "principle of induction." Indeed, if we eliminate the reference to time in the manner indicated by Mach, there appears to be complete identity between the above definition of Mr. Russell and the previous definition of Mach. It is, however, convenient to preserve the reference to time when we are considering the abstract science of mathematical physics. We consider an aggregate (which we may call the "space-time" aggregate) of real numbers in four dimensions which forms a numerical image (x, y, z, t) of our physical space and our physical time. This aggregate forms what we may call "absolute" space and time, and is presumably what Newton meant to indicate by certain well-known words of his. With this apparatus of space and time, the positions and motions of physical particles and bodies are represented by functions defined in the aggregate. The aggregate must be numerical if the motions are to be described by differential equations, for the same reasons that, as Mr. Russell pointed out in his *Principles*, the concepts of *differential coefficient* and *integral* imply *numerical* aggregates and not merely any ordered aggregates.

Since an "event" in abstract dynamics is a configuration (*Principles of Mathematics,* p. 486), the principle of induction is simply a problem of extrapolation; if we consider various particular values of an unknown function to be given and which represent various events,—such as the position of the center of a moving sphere as a function of time,—we have to seek a principle in virtue of which we can conclude the values of such a function at other times. It does not affect the nature of the principle if these other times are in the future or the past (cf. *Problems,* p. 101; *Ext. World,* p. 224). Of course, if the functions are quite general in their nature, no such inference can be made; so that if such inferences are to be possible, the functions in question must be of such a nature as to allow inferences from values given to values not given. Now, if a function of t is continuous and the values of the function for a certain infinite aggregate of values of t are known, we can conclude the values of the function at every point which is a limiting point of the t-aggregate, a part of the space-time aggregate just referred to. If the function t is differentiable, the same thing can be done; but if it is analytic, if we know its values for any period of time, however short, the mere knowledge that it is analytic enables us to conclude its value for any other value of t whatever. If, then, we have any reason to suppose that the functions which we assume to lie, as a subject for investigation, at the bottom of natural phenomena are of any special nature, this nature may enable us to give some definite information as to the form in which we can apply induction.

There is a doubtful point about all this: we assumed, in agreement with Mach, that the mathematical physics contemplated depends on functions which are at least theoretically known for all the values of the time; but it might seem that we can only know that nature is governed by law at all times by an application of the inductive principle (cf. *Problems,* pp. 98-103, 107). This objection was urged against me in a conversation of 1912 or 1913 with Mr. Russell, when I tried to explain the point of view of investigating the foundations of mathematical physics by determining the natures of the various functions used (cf. my notes in Mach's *C. of E.,* pp. 99-101). But it seems to me that there are two reasons against regarding induction as more fundamental than causality.

1. In the first place, mathematicians have become accustomed, at any rate since the times of Dirichlet, to regard the word "func-

tion" as meaning a correspondence between two aggregates (variables) even when the correspondence cannot be expressed by any known law. Thus, when we say that, for example, x is a function of t, we do not imply that, when t is fixed in value, the corresponding value of x can be calculated by a formula which expresses the law of correspondence: indeed, it can be established by simple arguments of which forms were used by Cantor as early as 1873, myself in 1903 and later, and Mr. Russell in *The Monist* for January, 1914, that there are functions in the general sense which cannot be represented as limits of infinite series of continuous functions. The importance of such reflections in this connection is that, when we say that x is a function of t, we do not imply that there is a formula correlating x and t; and that consequently we do not assume that x "depends" on t in the sense in which we would say that x "is given in terms of" t by a formula or "law of nature." It might be urged that such a function would be incapable of definition, but this would be a mistake, as is clearly shown by Cantor's method (1892) of defining uniquely a one-valued function which must necessarily be omitted from any one-one correlation of the arguments of this function with the class of one-valued functions possible for the same arguments, or by a simple application of my recent proof that any aggregate whatever can be well-ordered. It seems that Mr. Russell has been misled by his refusal to give ear to the teaching of the history of mathematics, and has paid the penalty of this refusal by repeating unconsciously the reasons which led D'Alembert, in the latter half of the eighteenth century, to maintain that the "arbitrary" functions which appear in solutions of certain partial differential equations obey the "analytic" law of being determined for the whole range of their arguments by the values for a small range of these arguments. In fact, in his formulation of the law of universal causation, Mr. Russell proceeds as follows (*Ext. World*, p. 221; cf. p. 219): "There are such invariable relations between different events at the same or different times that, given the state of the whole universe throughout any finite time, however short, every previous and subsequent event can theoretically be determined as a function of the given events during that time." That there is here an assumption that such relations are what Weierstrass understood by the words "analytic functions" can be proved quite simply by showing that we cannot,—excluding for obvious reasons the "monogenic" functions of Borel,—reduce the

states at all times from the states during some interval of time when the function is not analytic. Suppose the state were known to be constant throughout the finite space of time from a to b, including the ends: if the function were analytic, we could conclude that the state was constant throughout all time; but this need by no means be the case if the function were merely continuous or, say, merely differentiable.

2. I now come to the objection to founding causality, through the "principle of induction," on the notion of probability. The world of mathematical physics,—or at least one of such worlds,—is a schematic representation of reality, in which all causes which are not predominating ones are left out of account and treated, for the sake of saving the trouble of attending to many things whose influence is very small, by the calculus of probabilities. Thus, this calculus studies effects about the causes of which we are ignorant; and the usual (Laplace's) definition of "probability" implicitly contains a reference to an assumed non-existence of a predominating cause (cf. S. Pincherle, *Scientia*, Vol. XIX, 1916, pp. 417-426; cf. my account in *Mind*, N. S., Vol. XXVI, 1917, pp. 243-244). It seems, then, that it is when we try to decide, without making use of the notion of cause, which—if any—is the most probable of various configurations at time t' of a physical system that we meet those "terrible difficulties in the notion of probability" spoken of by Mr. Russell (*Ext. World*, p. 36). And, as an attempt to define causality in terms of probability is an attempt to walk in a vicious circle, we can hardly avail ourselves of the permission Mr. Russell went on to give, "—but we may ignore them for the present."

Let us try to make clear to ourselves what sort of questions we have to deal with in the calculus of probabilities. First of all, we may put on one side all assertions about the "probability" or otherwise of a known *proposition* q being implied by a known *proposition* p, the actual deduction not having been made, and confine discussions about "probability" to certain discussions about the truth-value of propositional *functions*. Now, although we may speak, say, of "the probability that $\phi\ (x)$ is true," where we know the form ϕ, the range of x, and the truth-value of $\phi(x)$ for any x in the range (it is m/n, where n is the number of x's in the range and m is the number of those x's for which $\phi(x)$ is true), it is not difficult to see that the characteristic questions arise only when one or more of the arguments are *unknown* coordinates of space or

time. Questions in probability are thus essentially questions in physics.

But it is a simple deduction from a known physical law that, of all paths which might be described by a system of known constitution (iron, brass,...) and connections under any forces (gravitational, magnetic,...), that path which is described by the system if it is a purely dynamical one (the given *masses* are relevant and not their constitution, such forces as that of gravitation are alone admitted, and "concealed" masses not assumed) is that determined by the rules of the calculus of probabilities to be the "most probable" one. In fact, Gauss observed at the end of the memoir (1829) in which he announced a new form of the fundamental law determining the motion of any dynamical system whatever: "It is very remarkable that the free motions [of the particles of the system under the given forces], if they cannot take place owing to the conditions of the system, are modified by nature in precisely the same manner in which the calculator compensates, according to the method of least squares, experiences (*Erfahrungen ausgleicht*) which relate to magnitudes connected of necessity with one another. This analogy can be still further followed out, but to do so is not my intention at present" (cf. the reprint of the original in No. 167 of *Ostwald's Klassiker*, p. 30, and my notes on pp. 46-47, 68; cf. also Mach, *M.*, pp. 350-364). This Gaussian "law of least constraint" determines uniquely the motion of the system when the mechanical forces, masses, connection, and initial conditions are known; and it seems that we may define "most probable" as "mechanically determinable." Everybody will at once recall such facts as our greater familiarity with those events which are of a predominantly mechanical nature; the surprise occasioned in us by a physical experiment, such as Oersted's, from which results a motion which cannot, without going beyond the sense-data, be explained mechanically (cf. Mach, *C. of E.*, p. 69); and that the law of the arithmetical mean for averages is a consequence of the law of least squares.

* * *

We will now consider the law of causality as an *a priori* principle.

In all natural science our aim is to complete facts in thought—whether for practical or purely intellectual ends or both. For this purpose we set up a model—a mathematical construction in thought, and so arrange that the logical consequences of premises in our

model should represent—approximately at least—the events which have very frequently followed certain other events in nature, while there are other consequences which represent what might be unobserved events. Thus our model might contain Galileo's formula ($\frac{1}{2}gt^2$) for falling bodies, which would give results for times, say in the next century, at which no observations have been made. We then presuppose that it is possible to complete facts in thought; if this supposition were not true, it would obviously be impossible to have any science at all, but only mere descriptions of isolated observations. The case is something like the case of the solution of differential equations in mathematics: solutions of certain more or less isolated kinds of differential equations have been known for over 230 years, but it was less than a hundred years ago that Cauchy showed that, logically speaking, the first question was to prove, in very general cases, the existence of solutions. He did this for many cases. Now, in physics we cannot prove logically the existence of events corresponding to our above-mentioned conclusions, as we could if physics were founded exclusively on logic; and consequently, from a logical point of view, we must *assume a priori* this existence *unless* we find it possible and desirable to deduce propositions from our model without any reason whatever for believing or disbelieving that they represent completions of facts. This last process is analogous to the practice, if it has ever been practised, of setting up functions which may be solutions of differential equations although we have no *a priori* reason for thinking so and no kind of assurance beforehand that solutions exist. It is under the assumption that we must know that it is possible to complete facts in thought that Mr. Russell (*Ext. World,* p. 223) says that "there must necessarily be some *a priori* principle involved in inference from the existence of one thing to that of another"; but he chooses, on the grounds that the formulation of the law of causality is complicated and its assumption *a priori* is therefore unplausible (*ibid.,* pp. 35, 223; cf. *Myst. and Logic,* p. 195), that the "principle of induction" is more fundamental. He rightly remarks (*Ext. World,* p. 36) that Mill's "method of simple enumeration" does not invariably give true results, and therefore discovers a way of saying something involving the method of simple enumeration that is invariably true,—its "probability" increases indefinitely with the number of instances. Apart from the introduction of an indefinable,—"probability,"—in a way that seems to appeal to popular rather than scientific minds, this

has the same meaning as: "The propositional function $\phi(x)$ is not true for all x's, but the propositional function 'for every x either $\phi(x)$ or not-$\phi(x)$' is invariably true."

The principle of causality may, it seems to me, be stated without any use of an indefinable "probability" as follows:

There is a one-one correspondence between any portion of the universe and the whole of the universe such that any change in the portion necessarily implies some change of all the rest. Of course it must be added that most of these changes are quite imperceptible to ordinary observation: what is meant is that, speaking with the utmost strictness, nothing in the world can change without everything else also changing. Imagine a person jumping on the surface of the earth: common sense would say at once that it is absurd to say that the jumper moves the whole earth. But if we consider the case of two gravitating masses m and n, of which m is very great and n is very small and in contact with m; and n is alive and jumps a short distance from the surface of m; then the common center of gravity of m and n remains fixed, and consequently the whole mass m moves a very minute distance. Such instances take away the appearance of paradox in the principle of causality just formulated; and we must always remember that, for practical purposes all very minute alterations may be disregarded. Thus, although strictly speaking the correspondence between the variations in any portion and the whole is one-one, for practical purposes we can regard the correspondence as many-one and say that changes may possibly take place without any (perceptible) alteration in most other things. This comes to the same thing as pointing out that these are "practically isolated systems" (cf., e. g., *Myst. and Logic*, pp. 197-198; *Ext. World*, p. 226). In the account given by Mach of the functional dependences of changes in any portion of nature or changes outside, these functions were, since Mach never concerned himself with mathematical refinements, many-one, and consequently he was forced to admit that there are, as there appear to be, cases (of which an example was given in *M.*, p. 503) in which a certain phenomenon B can vary without a corresponding variation of the phenomenon A, although to different A's correspond different B's. It is to be noticed that Mach rejected, as everybody concerned with practical work does reject, the notion of a change determining a change of the *whole* universe.

It is to be remarked that Mach expressed himself very strongly

as to the absence of meaning in such a phrase as "the state of the universe." At least he did this in his publication of 1872 (*C. of E.*, pp. 62-63),—but he did not repeat his objections in his *Mechanics*. This may have been due, either to his supposition that the correlation between changes in a portion and in the whole is many-one, or to his definition (*C. of E.*, p. 63; *M.*, p. 502) of "the cause" of A as the *totality* of sense-data which condition the existence of A, and he may have suspected that such a cause may involve the whole universe. But it seems that we might speak of a propositional function involving x, where x is *anything* in the universe, without speaking of the universe as a subject of the propositional function. Further, it seems true that we cannot speak of a space and time which have existences apart from the universe itself; for "space" and "time" seem to be merely convenient ways of describing the relations of the universe to any particular event. But, in our mathematical schemes of the world, we introduce absolute space and time, and, though we might seem to be able to consider the universe in absolute space and time, we find on closer consideration that our schemes are merely approximations to a representation of a portion of the universe which is a practically isolated system. The contrast between the absolute space and time of mathematical physics and the relative space and time of physics will be more fully developed on another occasion; here we are only concerned with the facts that strictly speaking the cause of any effect is the whole universe; and, on the other hand, that it is always possible—often it would seem very simple—to regard for practical purposes physical systems in a small portion of the universe as isolated. It is not asserted that there is an *a priori* principle of causality: all that is asserted is that, *if* there are "laws of nature," they are *a priori*, for it is, in general, logically impossible to determine a law from a finite number of observations,[3] and "probability," even if it could—which would seem doubtful—serve to bridge the gap between observation and law, cannot, without a vicious circle, be used to define "causality," but can itself be defined by means of a certain causal law. Though we may have to assume that there are "laws of nature," we cannot really prove, except by some further hypothesis, that, for instance, the "law of gravitation" is such a "law."

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[3] If we knew somehow that the law is an *algebraic* function, this would be possible; if it is not, an infinity of observations would always be necessary.

II.

Dr. Charles A. Mercier (*On Causation, with a Chapter on Belief,* London: Samuel George, 52a High Holborn, W. C. 1; price 7s. 6d.; pp. xii, 228) has published a book in which he attempts an investigation into the problems of causality and belief. In the first part of the book, Dr. Mercier deals critically with various theories of causality. In the second part, he expounds his own views concurrently with criticizing Mill's views. In the last part of the book the fascinating subject of the nature of belief is dealt with.

In the course of his criticisms, Dr. Mercier gives short and therefore necessarily inadequate accounts of the views of Hume, Mr. Welton, Prof. Karl Pearson, Mr. Bertrand Russell, and Dr. Mac Taggart. Sympathetic understanding seems to be the basis of all really valuable criticism. It is lack of this sympathetic understanding that makes so much of the criticism in this volume disappointing. Particularly in the treatment of Mr. Russell's theory of cause is this defect plain. The outcome (p. 34) of the critical examination of the theories is interesting and must be quoted:

"In conclusion, it is suggested that the inability of philosophers to define causation in consistent and intelligible terms argues, not that causation is imaginary, but that philosophers are incompetent."

It may be worth while to examine more closely Dr. Mercier's criticism of the theory of causality put forward by Mr. Russell. Mr. Russell has carried his discussion of causality farther than most philosophers, and has stated clearly various difficulties in the traditional formulation of the principle. Thus, for example, in the article "On the Notion of Cause" (*Mysticism and Logic, and Other Essays,* London, 1918), the question of the time-interval between cause and effect is discussed. It is explained that cause and effect cannot be contiguous, for "no two instants are contiguous, since the time-series is compact." Mr. Russell has defined on various occasions what he means by "compactness" (cf. *The Principles of Mathematics,* London, 1903, p. 271, and *Our Knowledge of the External World,* Chicago and London, 1914, p. 132). A series of terms is "compact" when no two terms are consecutive. Contiguous terms are consecutive. Thus it is not correct to say, as Dr. Mercier says (p. 25), "the more compact the time-series, the more closely contiguous must be its instants." It is interesting to see an example here of the kind of alteration the modern philo-

sophical point of view, which is based on logic, has made in the treatment of certain problems. With the technical apparatus of modern logic, it has been found possible to offer an analysis of the nature of such a series as the time-series appears to be. With this analysis, a further discussion of the problems of the time-relation between cause and effect can be given, and the difficulty of allowing that cause and effect are contiguous in time, or are separated by a finite interval in time, has been made clear. For it can only be urged that cause and effect are contiguous in time if it is assumed that the time-series is discrete and therefore discontinuous, and this very few thinkers appear to assume. On the other hand, if there is a finite interval between the cause and the effect, there is always the possibility of something happening during the interval to prevent the expected result. Thus, when I put a penny in the slot, an earthquake may ensue before I can get out my ticket, and thus putting the penny in the slot is not, by itself, adequate to ensure the effect. All this discussion Dr. Mercier sets aside as (p. 26) "having no relation whatever to causation, as it is known on this earth." But people do put pennies in slots, and things might happen to prevent the tickets coming out.

It is to be remarked that Dr. Mercier himself has not seen the difficulty in question, for in talking of a man pushing a trolley (p. 26), he says that "the effect begins as soon or almost as soon as the cause begins." Here is the very difficulty which Mr. Russell is discussing. And thus, Dr. Mercier not only does not dispose of Mr. Russell's contentions by calling them (p. 26) "pretty word-spinning," but actually, on the same page, introduces the identical difficulty without noticing that it *is* a difficulty, and therefore of course without discussing it. Dr. Mercier quotes Mr. Russell's statement of the principle of causality, and says (p. 27) that "by no perversity of ingenuity" could anything which can be discovered by means of it "be twisted or tortured into a cause." But in Mr. Russell's view (*Mysticism and Logic*, p. 180), "the word cause is so inextricably bound up with misleading associations as to make its complete extrusion from the philosophical vocabulary desirable." It is therefore scarcely to be expected that his statement of the principle would give us a way of discovering the cause of anything. Mr. Russell is engaged in formulating a principle which shall be a principle of causality in virtue of its fulfilling the condition of being a principle by means of which "from a sufficient number of

events, at a sufficient number of moments, one or more events at one or more moments can be inferred" (*Principles of Math.*, p. 478). In saying (p. 27) that "it is clear that no cause of any thing ever has been discovered or ever can be discovered, for we can never know the state of the whole universe," Dr. Mercier seems to be overlooking Mr. Russell's remark (*Mysticism and Logic*, p. 187) that "it must, of course, be admitted that many fairly dependable regularities of sequence occur in daily life" and (p. 192) "we may admit that, if any such sequence has been observed in a great many cases, and has never been found to fail, there is an inductive probability that it will be found to hold in future cases." This allows the possibility of our being able, with some degree of assurance, to detect casual sequences.

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In Chapter II the constructive part of the work is begun and the relation involved in causation is examined. The following definition is reached (p. 47): "A cause is an action or cessation of action connected with a sequent change or accompanying unchange in the thing acted on." An "unchange" (p. 48) is an effect "which impresses us as....the prevention of change." In Chapter III there is a discussion of condition, and in Chapter IV an examination of (p. 77) "the five characters or marks that are said to be characteristic of causation, viz., equality of cause and effect, immediacy, unconditionality, invariability, and antecedence"; which "shows that not one of them properly or necessarily pertains to causation." The definition of causation is modified and finally stands as (p. 77) "the necessary connection between an action and the sequent change or accompanying unchange in the thing acted on." In the next chapter the doctrine of Plurality of Causes is attacked and Mill's work on it reinterpreted. For the Law of the Uniformity of Nature, Dr. Mercier proposes to substitute an "Axiom of Causality" (p. 101): "Like causes in like conditions produce like effects, and the more closely alike the causes and the conditions, the more closely alike will be the effects." In Chapter VI, twelve methods of ascertaining causes are given. These methods (p. 146), "used by scientific men in scientific matter, are precisely the same as those used by every one else in the common affairs of daily life." Finally a list of the three things which it is necessary to establish in order to prove causation are given (p. 165):

1. Action on the thing on which the effect is produced;

2. Precedence of the action on the change, or accompaniment of the action with the unchange;

3. Connection between the action and the effect, and a list of common "blunders" is added.

Chapter VIII, which deals with the causes of death and the causes of insanity, is absorbing and original and provides interesting examples of Dr. Mercier's methods of inquiry.

* * *

The last part of the book deals with belief. Now, in elaborating a theory of belief, it is essential first to find as many propositions about it which are obviously true as one can. Dr. Mercier does not get to any definite theory of belief, but he points out many interesting things about it which are relevant, and must necessarily be taken into account if a satisfactory theory is to be established. It is, therefore, in this preparatory stage of the work of solving the problem of belief that Dr. Mercier's contribution is most helpful.

It is pointed out that there is an ambiguity in the use of the term belief, that it may mean "being convinced of" or "being sure of"—as, for example, in "I believe there is a God" and "I believe it will keep fine." It is, of course, the latter usage, in which believing and knowing can be contrasted, that it is interesting to analyze, and it is this concept which Dr. Mercier proceeds to discuss. Belief, Dr. Mercier suggests, is an attitude of mind toward a fact or toward a statement about a fact. This statement would be more interesting if the view adopted as to the concept of mind were explained. To say that belief is an attitude toward a fact or toward a statement about a fact seems to mean that an act which is a believing that a is to the left of b is of the form:

x has a certain relation to (a being to the left of b), or

x has a certain relation to S (a being to the left of b) where S (a being to the left of b) is a statement about (a being to the left of b).

Now there are false beliefs. If this particular belief is false, a will not, in fact, be to the left of b and so there will be no fact which can be described by "a being to the left of b." This difficulty Dr. Mercier clearly feels and it is because of it that an alternative of a relation to a statement about the fact is given. But the interesting point, under these circumstances, is to analyze what can be meant by a statement about a non-existent.

Dr. Mercier next points out the relational character of facts.

He is careful to mention that there are many relations which can subsist between terms, since, he says (p. 192), "the teaching of every book on logic is that there is only one relation between things, and that there is only one verb in any language, namely, the verb 'to be.'" But with this statement as to the teaching of "every book on logic," every student of modern logic is bound to quarrel. The doctrine that there is only one relation which can subsist between terms is certainly (p. 192) "a curious superstition, and well worthy the attention of students of irrational beliefs." The falseness of this doctrine has, of course, been generally recognized by modern logicians, but it is perhaps worth while pointing it out again. But it is a curious mistake on the part of Dr. Mercier to imagine he is making a discovery in pointing out that it is false. The point as to the relational character of facts is again a familiar one to all except those who are entirely ignorant of modern logic,—but the suggestion as it stands is too narrow. It is stated (p. 192) that "our expression of a fact is always in the form 'A is related to B,' and this empty form is filled out and vitalized by substituting appropriate terms for A and B, and by interposing between them a verb as a connecting link, as for instance, 'Hens lay eggs.' This is an expression of a fact, and the fact is expressed by asserting a relation of laying, which means...parentage, between the eggs and the hens." Now we cannot accept this very simple interpretation of the form of the fact "Hens lay eggs," and it seems not extravagant to suppose that there are facts not expressible in the form:

A is related to B.

Thus, in geometry, we have such facts as:

A is between B and C.

Such a fact is relational, but the relation subsists between three terms. It is needless to multiply instances, but modern logic has not at present been shown to be wrong in its theory of the existence of relations between more than two terms. Dr. Mercier's assertions cut out immediately any such theory. By his dictum on p. 217: "Whoso makes an assertion, upon him lies the burden of proof" (if there is anything in such a principle, which seems doubtful), it is for him to prove that all facts are of the dual relational form. Such an assertion most definitely contradicts the views of the modern logician and must be challenged.

In considering various beliefs, Dr. Mercier points out that some statements are not merely false, but definitely nonsensical and such

that the mind will not even consider them. Such are, for example, "Two o'clock is solid," "Limestone reasons downward." That beliefs are never nonsensical is an important, if obvious point to be remembered in constructing a theory of belief. The notion of nonsensical as employed by Dr. Mercier is, of course, the same notion as that of "non-significant" which is used in modern logic.

But there are also beliefs which, though not nonsensical, are instantly rejected by the mind. This class are clearly those which contradict the laws of logic. The examples given, "The hen laid an egg larger than itself," "The solid body is liquid," at once contradict definite logical propositions.

Dr. Mercier asserts in the course of his discussion (p. 227) that "belief ought to conform to fact, but cannot be directly related to fact, for we have no direct knowledge of fact. Between belief and fact there is always the intermediary of evidence." Evidence, it appears, is of three kinds—evidence of sense, evidence of reason, evidence of hearsay. Now (p. 193) "when we have an impression on a sense, when we see a light, hear a sound, or feel a touch, these are facts of ultimate certainty; and it is not open to us to doubt that we do experience the sensation." But, in allowing that we cannot doubt that we have certain sensations, is not Dr. Mercier contradicting his previous assertion that we have no direct knowledge of fact? "A fact" (p. 225) means "anything existing or happening in the past, present, or future." Thus the having of a sensation is clearly a fact, and one which we cannot doubt. Further, how will Dr. Mercier treat our knowledge of the truth of logic? Some of our beliefs in logical proposition are not based on anything. Our knowledge of the law of contradiction is, probably, direct. Anyhow our knowledge of some of the propositions of logic is direct.

In his treatment of "empirical certainty," Dr. Mercier makes some assertions which seem to be difficult to justify. He states (p. 227): "That relation which has been found constant....in experiences diverse and incalculably numerous, is true for us." Further (p. 228), "If the relation is not constant in experience, then the degree of belief ought to correspond with the proportion that the positive instances in experience of the relation bear to the negative instances in which the terms of the relation occur apart." But on p. 207 another suggestion is made. We have as before: "If these experiences are sufficiently numerous, and are all one way,

we must believe that the experience is necessary, and will continue...." But also: "When experiences are not constant...we are compelled to believe that a similar proportion will hold of such experiences in the future." This is a different proposition from the other. Neither as it stands is satisfactory. They employ words involving all kinds of difficult points (e. g., "sufficiently numerous," "necessary"). The immense difficulties and subtleties of the problem are not even mentioned. It would be almost impossible to deal satisfactorily with all the vital questions which are involved in the small space left to Dr. Mercier for this discussion, but we venture to think that the treatment of the subject suffers from the absence of any hint as to the view taken of probability. The relation of probability to induction seems one of the most baffling problems. Its position is fundamental in any inquiry such as Dr. Mercier's.

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Dr. Mercier's treatise is throughout fresh in its treatment and keen and practical in its point of view. The taunts at philosophers and logicians, though they occur so very many times that one tires of their biting humor and sharp wit, are not essential to the book. One cannot help feeling that a more intimate acquaintance with the work of the modern logicians would cause Dr. Mercier to modify his opinions and change his exceedingly ungracious attitude to philosophers and thinkers generally, which he takes every opportunity of expressing in the present work.

DOROTHY WRINCH.

LONDON, ENGLAND.

III.

Miss Wrinch quotes me as formulating an "Axiom of Causality," a thing I have never done, and should never think of doing, for if causality means something different from causation, I do not know what it means, and if it means the same, I object to calling the same thing by two names, and thus introducing confusion and preparing the way for a fallacy of bivocation.

For the rest, it would be profitless for me to follow Miss Wrinch point by point; for the difference between us is fundamental, and no discussion could diminish it. She regards causation, or causality, as she prefers to call it, from the standpoint of philosophy, as a matter of words, or at the utmost of thoughts, for philosophers to wrangle over. I look upon it from the standpoint of science, as a

matter of actual occurrences in the world of experience, occurrences whose nature is to be explained, whose occurrence is to be investigated, identified, and accounted for. I assume that is a thing or event that actually happens in the world of experience. Miss Wrinch follows Professor Pearson and Mr. Bertrand Russell in assuming that it is a word that can be banished from philosophical discussion, and will then cease to exist. There can be no reconciliation between these two attitudes of mind—I apologize for this expression, which Miss Wrinch objects to but which will, I think, be generally understood. If I were to traverse her criticisms, I should be belaboring what seems to me to be shadows, and entering upon a profitless verbal wrangle. I trust Miss Wrinch will not think me discourteous if I say that, living as I do in a world of realities, I have something better to do.

CHAS. MERCIER.

PARKSTONE, ENGLAND.

IV.

I would like to add a little to Dr. Mercier's reply to my criticism. But, first, I must apologize for calling his "Axiom of Causation" an "Axiom of Causality" by mistake.

It is a surprise to me that Dr. Mercier thinks that the difference between his attitude and mine is fundamental. My attitude was merely that of impartial inquiry. When he criticizes the work of great thinkers in a way which seems to me to miss all the important points, it is, I feel, time to lodge a protest. But I am at a loss to understand how this can enable Dr. Mercier to deduce anything about my own opinions—or how they are relevant to the discussion. If my view is of any interest to him—and I could never have guessed that it would be—I may say that the questions which Mr. Russell has discussed in his work on causation do not seem to me to be shadowy, neither do they seem to me to turn on verbal points.

All the criticisms which I have made remain unanswered. In particular it is to be regretted that Dr. Mercier has not seen fit to deal with the points I brought up in connection with modern logical theory, which seems to me to be of vital importance in such subjects as causation and belief. A general and wholesale condemnation of philosophers seems hardly the best way to meet criticism.

DOROTHY WRINCH.

LONDON, ENGLAND.

CURRENT PERIODICALS.

In *Scientia* for January, 1919, the article on Greek influence in the development of Hindu mathematics, by G. R. Kaye, may not unlikely be a grievance against the oppressor, in that it displaces a lustrous jewel in the crown of Hindu culture. It challenges the hitherto undisputed dictum that with no aid, or with little outside aid at all, the inhabitants of the great peninsula attained considerable distinction as exponents of mathematical science. Kaye now congratulates mathematical historians on the scepticism with which they have regarded these assertions. He is cruel enough to present claims and conclusions in the simple forms: the Hindus invented our notation (probably untrue); and became the greatest calculators of antiquity (does not entirely agree with the facts); as certain propositions are for the first time met with in Hindu works, therefore the Hindus must have discovered them (pure romance). The author examines one by one the partisans of the Hindu view, demolishes or modifies most of their assertions, and then proceeds along lines of his own choice. He finds every proposition in the Hindu works of a period in the same or slightly varying form in the works of the Greeks, and nowhere in the oldest Hindu literature. The ancient Hindu mathematicians themselves are shown to have looked down with scorn on their predecessors. Brahmagupta, for instance, claims no originality and even indicates his sources of information. Kaye seems to have hopes that some day or other we shall discover a copy of the lost works of Diophantus, or of Hypatia. The matter will then be at rest.—Sir Oliver Lodge takes "Ether and Matter" once more as his subject. He traces the steps by which electricity came ultimately to be considered as the raw material out of which matter is composed, how the material atoms may be explained with the help of electrons, how electric charges are composed of modified ether, how we may regard all kinetic energy as belonging to matter whether atomic or corpuscular, and all static energy to the unmodified and universal ether. Coexistence and interaction form the fundamental joint quality of matter and ether. "The potentiality of interaction, and often the conspicuous reality of it, everywhere prevails and constitutes the whole of our purely mundane

experience."—G. Levi takes as his topic the life of the isolated elements of the organism. He describes the work of Harrison, Born, Giardina, W. and M. Lewis, and congratulates morphology on its possession of a new technique. "For the first time we are able to analyze the structure of the living protoplasm of higher organisms, to follow its transformations, and to determine the behavior of protoplasm under artificial stimuli." He believes that the culture methods of investigation will have a decisive influence upon the future of investigations upon the structure of cells and tissues.—P. Bellezza's note on the phonology of the wider Rome, of Romania, explains how it is proposed to trace the determining causes of the evolution products of the transformation of the Latin tongue in the various countries through which it spread and exercised its potent influence. For this purpose it is necessary to summon to our aid the assistance of more than one science of which the older grammarians and etymologists had never heard or dreamed. The laws of phonology have their physiological basis. Phonetic phenomena are produced and remain identical as long as the conditions remain unchanged. The absolute uniformity of such laws continues until some psychic element intervenes, such as is afforded by contact with races speaking other tongues. This brings us to the "dead point" of phonetic laws. By the combined aid of physiology, ethnology, geography, and history, it is hoped that considerable light will be thrown upon questions that have perplexed generations of students of linguistics.

In *Scientia* for February, 1919, José M. Plans writes on the introduction of the method of perturbations into general mechanics. Work in this direction is associated with the names of Kobb, Moulton, and Behrens. He indicates how in a limited number of cases the classical method of celestial mechanics may be applied to general mechanics.—Alexandre Moret writes on Egyptian hieroglyphics, a subject which has enthralled many men of his race from the days of Champollion.—E. Claparède discusses the new educational principles which are to transform our schools. The old system under which the social instincts were suppressed is doomed: we are to have democratic institutions in which internal discipline takes the place of a discipline imposed from without, the key-note being "self-government." We shall have no teachers who are not psychologists, and when we say that the children of the future are to do nothing but what they like, perhaps it should be paraphrased into: they

will do with all their heart what they are at because they will want to do it.

In *Scientia* for March, 1919, we are reminded that J. L. E. Dreyer has a prescriptive right to the handling of any new material concerning Tycho Brahe. His article on the place of the famous astronomer in the history of his science is no doubt suggested by a discovery made in the course of preparing a new and complete edition of the manuscript books of Tycho's nightly observations. The discovery refers to what has always been "an extraordinary riddle." What the puzzle is may be found in the article, to which we refer the reader.—Luigi de Marchi describes the various methods, deduced from general principles, which are used for representing the surface of the earth. The various types of projections are carefully explained and their relative values indicated. From these he passes to those types which combine the merits of two or more of the normal types, and involve the use of artifices and constructive adaptation, e. g., "polyconical projections, on one of which is constructed the English War Office map."—Ingvar Jörgenson and W. Stiles discuss the present condition of the problem of the assimilation of carbon—a matter, they claim, of general interest to humanity, seeing how the solution must affect human needs, from the point of view of plant physiology and, when the time comes, from its connection with the utilization of the radiant energy of the sun. A careful account is given of recent research on the subject, and as the general outcome of their critical examination of existing claims they conclude that there is still "a very long row to hoe" before success can be said to be in sight.—L. Houllevigue discusses the future of pure science. In his own country he foresees that money, men, and methods, all of which are essential for the future of pure science, will go to the development of the industries. He has no panacea to offer, but he closes his pessimistic outlook with a call for money enough to keep body and soul together in those engaged in pure research, for a careful selection of the right men to carry on the good work, and for the systematic organization of the work of the future.—R. Assagioli calls the attention of European psychologists and alienists to the work of Morton Prince and Boris Sidis, and to the discussions between them and the disciples of Freud, which have appeared in the *Journal of Abnormal Psychology* and elsewhere.

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BOOK REVIEWS AND NOTES.

ESSAYS IN SCIENTIFIC SYNTHESIS. By *Eugenio Rignano*. Translated by *W. J. Greenstreet*. Pp. 254. London: George Allen and Unwin, Ltd.; Chicago: The Open Court Publishing Company, 1918. Price 7s. 6d. net.

"The need that is making itself felt in science for unification and synthesis between the individual sciences is extending to all branches of civilization and even to civilization itself. Analysis, no doubt, is essential to the development of knowledge, but practised by itself it inevitably leads to the creation of water-tight compartments, unless subordinated to the final process of red-integration in the corpus of human knowledge." The world is ruled by ideas. We have already learned the futility of maintaining water-tight departments within certain branches of science. We are beginning to learn that the so-called "overlapping of frontiers" between certain branches of science is really a misnomer, for with the extension of knowledge we find that such frontiers exist merely as conventions which no longer have the excuse of being thought convenient. For instance, we read in the number of *Science Progress* from which the first two sentences of this notice are taken that the future progress of geology imperatively demands an amalgamation with the other sciences, or rather, perhaps, a recognition of the fact that the study of the earth includes in itself all the sciences. From cases like this, which may be multiplied, it is not far to the idea that there is a sufficient body of material common to all the sciences to justify the view that there is room for the advent of a new type of worker—the theorist pure and simple. As compared with the specialist in any department of science he will be in certain respects at a great disadvantage: but on the other hand he will be unhampered by much that prevents the specialist from shaking off the attitude of mind which is necessarily the outcome of a life of work spent in contact with the concrete.

M. Rignano's conception of a theorist whose width of reading and grasp of results achieved shall inspire him to creative effort in the detection of unsuspected analogies, in the framing of hypotheses, and in the opening up of wider horizons, is as attractive as it is suggestive. Perhaps in one direction he has been partly anticipated by the enterprise of Dr. George Sarton, whose efforts to place research in the history of science upon a proper footing have been so lamentably frustrated in Europe by recent events, but who, we are glad to say, is already reaping some measure of success in America. Dr. Sarton points out in a paper in *Scientia* on "The New Humanism" how a just appreciation of the signification of science, and of relative values, must be based on a knowledge of its history, and that at the same time that knowledge will serve to correct in the specialist the narrowness of outlook to which he is inevitably exposed from the very nature of his work, will confirm him in the disinterested course which he has adopted, and will go far to check every tendency either to empiricism or to prejudice.

It is natural that M. Rignano's suggestions should be exposed from the

first to the charge of being too vague to be worthy of consideration. This charge has been anticipated by our author, who in this series of essays sets forth in the first place his reasons for believing his views to be feasible and necessary, and again how the work of the theorist may be done in the fields of biology and other allied natural sciences with the same prospect of success as has attended the so-called "intrusion" of mathematics into the field of physics. The author has with conspicuous effect explained his position, and the skill with which he has handled his subject is as remarkable as what will seem to many the audacity and the novelty of his views. In France and in Italy he has had nothing to complain of in the warmth of their reception, and as he claims that he has, after all, merely followed the example of some of the most famous English writers of the past, he should at any rate be rewarded by a careful examination of proposals which seem to so many of us to be fraught with hope for the future of science and humanity. γ

THE NEW TESTAMENT. Translated from the Sinaitic Manuscript, discovered by Constantine Tischendorf at Mount Sinai. By *H. T. Anderson.* Cincinnati: Standard Publishing Co., 1918.

By comparing this translation with the photograph of the Greek manuscript, the reader will discover two things:

1. Important matter omitted by the manuscript is added by the translator.

2. Important matter added by the manuscript is omitted by the translator.

Under No. 1 we refer especially to the Mark Appendix and the Adultery section in John. The Sinaitic manuscript ends Mark thus:

They said nothing to any one, for they were afraid of..... Gospel according to Mark.

The English reader wants to see just how this venerable document looks in Greek. Consequently the addition of the second-century Appendix ("Mark" xvi. 9-20) falsifies this entirely. So, too, with the woman taken in adultery in John viii. The manuscript passes the story over in silence, but the translator inserts it with a note. The Bible Society does the same, both with Mark and John, in its edition of the Armenian Version, another of the ancient witnesses against these Gospel additions. Granville Penn, the grandson of William Penn, was the first to print Mark in English as it appears in the oldest sources, ending abruptly at xvi. 8 (London, 1836). That lay scholar rightly classed the familiar editorial addition as apocryphal.

Under No. 2, the Epistle of Barnabas and the Shepherd of Hermas follow John's Revelation in the manuscript, without any note to indicate that they do not belong in the New Testament. This is just the kind of fact which the serious reader wants to know, but the translator withholds the knowledge. The New Testament, in the fourth century, was not yet sharply cut off from the Apostolic Fathers, not until 397.

The translator died many years ago and represented a now extinct school of theological shuffling. The Standard Publishing Company of Cincinnati has done very wrong to perpetuate this sort of thing. In science, public opinion has always required literary men to speak the truth. In the twentieth century it expects religion to do the same. ALBERT J. EDMUNDS.